FRM Part I Book 4: Valuation and Risk Models

Valuation and Risk Models

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FRM PART I BOOK 4: VALUATION AND RISK MODELS

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The following is a review of the Valuation and Risk Models principles designed to address the AIM statements set forth by GARP®. This topic is also covered in:

BOND PRICES, DISCOUNT FACTORS, AND ARBITRAGE

Topic 36

EXAM FOCUS

This topic provides an overview of the fundamentals of bond valuation. The value of a bond is simply the present value of its cash flows discounted at the appropriate periodic required return. Discount factors are used for pricing coupon bonds and for determining whether bonds are trading cheap or rich. If a mispricing exists among securities, a riskless arbitrage profit can be made from the violation of the law of one price.

AIM 36.1: Describe and contrast individual and market expressions of the time value of money.

Professor's Note: To help answer AIM 36.1, recall the time value of money principles presented in Book 2. A dollar today will be worth more in the future given a positive rate of return. Similarly, a dollar paid in the future will be worth less in present value terms.

DETERMINING BOND CASH FLOWS

Before computing the value of a bond, first determine the appropriate cash flows. The cash flow of a coupon bond has two components: periodic coupons and par value at maturity. Coupons are often stated in annual terms and must be adjusted for periodicity. Once the cash flows are determined, the present value (PV) of the cash flows can be computed.

Example: Bond cash flows

Suppose we have a \$1,000 par value, 10-year bond that pays a 10% coupon semiannually. **Determine** the cash flows of this bond.

Answer:

There are 20 semiannual payments of $\$50 = \left(\frac{0.10}{2} \times \$1{,}000\right)$. A lump sum payment of \$1,000 occurs at the end of the 20th period (in 10 years).

Fundamentals of Bond Valuation

The value (or price) of any financial asset—such as a bond—can be determined by summing the asset's discounted cash flows. There are three steps in the bond valuation process:

1. *Estimate the cash flows.* For a bond, there are two types of cash flows: (1) the annual or semiannual coupon payments and (2) the recovery of principal at maturity, or when the bond is retired.

2. *Determine the appropriate discount rate.* The approximate discount rate is either the bond's yield to maturity (YTM) or a series of spot rates.

3. *Calculate the PV of the estimated cash flows.* The PV is determined by discounting the bond's cash flow stream by the appropriate discount rate(s).

As previously noted, bond investors are entitled to two distinct types of cash flows: (1) the periodic receipt of coupon income over the life of the bond and (2) the recovery of principal (or par value) at the end of the bond's life. Thus, valuing a bond deals with an *annuity* of coupon payments plus a large *single cash flow*, as represented by the recovery of principal at maturity, or when the bond is retired. These cash flows, along with the required rate of return on the investment, are then used in a present-value-based bond valuation model to calculate the dollar price of the bond.

In this topic, the concentration is on bonds that pay coupons semiannually in even 6-month intervals from settlement.

Price Quotations

Bonds are quoted on a percentage basis relative to a par value (100). Bonds priced at par are quoted at 100; bonds sold at a discount are priced at less than 100; and bonds sold at a premium are priced at greater than 100. U.S. Treasury notes and bonds use a "32nds" convention. A bond quoted as 97-6 (or 97:06 or 97.6) is interpreted as $97\,{}^{6}\!/_{32}\%$ of par value. If the par amount is \$1 million, the price of the bond is 97.1875% of the par amount, or \$971,875.00. Corporate and municipal bonds are quoted in eighths (e.g., $102\,{}^{1}\!/_{8}$ indicates the price of the bond is 102.125% of par). In either case, convert a price quotation into a dollar price simply by adding the decimal equivalent of the fraction to the base number of the quote and using that as a percentage of the par amount to compute the price.

A "+" in the quote indicates a half tick. For example, if the price is quoted as 101-12+, then the bond would sell at $101 + {}^{12.5}\!/_{32}$.

Discount Factors

AIM 36.2: Define discount factor and use a discount function to compute present and future values.

Discount factors are used to determine the present value. The discount function is expressed as $d(t)$, where t denotes time in years.

Example: Calculating bond value using discount factors

Suppose that the discount factor for the first 180-day coupon period is as follows:

$$d(0.5) = 0.92432$$

Calculate the price of a bond that pays $108 six months from today.

Answer:

Since $1 to be received in six months is worth $0.92432 today, $108 received in six months is worth 0.92432 × $108 = $99.83 today.

Professor's Note: The future value of $1 invested for time t is 1/d(t).

Bonds are securities that promise a future stream of cash flows, so a series of Treasury bond (T-bond) prices can be used to generate the discount function.

Example: Calculating discount factors given bond prices

Figure 1 shows selected T-bond prices for semiannual coupon $100 face value bonds.

Figure 1: Selected Treasury Bond Prices

Prices are from 5/14/06, with t + 1 settlement			
Bond	*Coupon*	*Maturity*	*Price*
1	4.25%	11/15/06	101-16
2	7.25%	5/15/07	105-31+
3	2.00%	11/15/07	101-07
4	12.00%	5/15/08	120-30
5	5.75%	11/15/08	110-13+

Generate the discount factors for the dates indicated.

Answer:

Bond 1:

When this bond matures on 11/15/06, it makes its last interest payment of

$2.125 = \left(\frac{0.0425}{2} \times \$100\right)$ plus the principal repayment of 100. The present value of the 102.125 is given as the price of 101-16, or 101.50.

price(PV) = CF(0.5) × d(0.5)

101.5 = 102.125 d(0.5)

Solving for the discount function yields:

d(0.5) = 0.9939

Moving farther out on the curve, the function becomes slightly more complex, as each point of the curve must be included. For example, to solve for Bond 2, we must include *d*(0.5) as well.

Bond 2:

The coupon payment at time 0.5 is 3.625 = 7.25 / 2. The final cash flow at time t = 1 is 103.625 = 100 + 3.625. Those two cash flows discounted back to 5/14/06 using the discount function should equal the price of the bond:

105-31+ = 105.9844 = 3.625 d(0.5) + 103.625 d(1)

Since it's already known that d(0.5) = 0.9939, substitute that value into the equation and solve for *d*(1):

105.9844 = (3.625 × 0.9939) + [103.625 d(1)]

d(1) = 0.9880

Using the same methodology for Bonds 3, 4, and 5:

Bond 3:

$$101.2188 = 1.0\ d(0.5) + 1.0\ d(1) + 101\ d(1.5)$$

Thus:

$$d(1.5) = 0.9825$$

Bond 4 and Bond 5:

$$d(2.0) = 0.9731$$

$$d(2.5) = 0.9633$$

Figure 2: Discount Factors

Time to Maturity	*Discount Factor*
0.5	0.9939
1.0	0.9880
1.5	0.9825
2.0	0.9731
2.5	0.9633

Determining Value Using Discount Functions

AIM 36.3: Define the "law of one price," support it using an arbitrage argument, and describe how it can be applied to bond pricing.

AIM 36.6: Identify arbitrage opportunities for fixed income securities with certain cash flows.

The discount functions previously mentioned can be used to estimate the value of a bond. Since investors do not care about the origin of a cash flow, all else equal, a cash flow from one bond is just as good as a cash flow from another bond. This phenomenon is commonly referred to as the **law of one price**. If investors are able to exploit a mispricing because of the law of one price, it is referred to as an arbitrage opportunity.

Example: Identifying arbitrage opportunities

Suppose you observe the annual coupon bonds shown in Figure 3.

Figure 3: Observed Bond Yields and Prices

Maturity	*YTM*	*Coupon (annual payments)*	*Price (% of par)*
1 year	4%	0%	96.154
2 years	8%	0%	85.734
2 years	8%	8%	100.000

The 2-year spot rate is 8.167%. Is there an arbitrage opportunity? If so, describe the trades necessary to exploit the arbitrage opportunity.

Answer:

The answer is yes, an arbitrage profit may be realized because the YTM on the 2-year zero coupon is too low (8% versus 8.167%), which means the bond is trading *rich* (the bond price is too high). To exploit this violation of the law of one price, buy the 2-year, 8% coupon bond, strip the coupons, and short sell them separately. The discount factors are derived from the prices of the zero-coupon bonds.

Figure 4: Discount Factors

Time to Maturity	*Discount Factor*
1.0	0.96154
2.0	0.85734

To demonstrate the process of exploiting the arbitrage opportunity here, consider the following 3-step process (the dollar amounts given are arbitrary):

Step 1: Buy $1 million of the 2-year, 8% coupon bonds because they are trading *cheap*.
Step 2: Short sell $80,000 of the 1-year, zero-coupon bonds at 96.154.
Step 3: Short sell $1.08 million of the 2-year, zero-coupon bonds at 85.734.

Figure 5: Cash Flow Diagram

Time = 0		*1 year*		*2 years*	
–1,000,000.00	(cost of 2-year, 8% coupon bonds)	+80,000	(coupon, interest)	+1,080,000	(coupon, interest)
+76,923.20*	(proceeds 1-year, 0% bonds)	–80,000	(maturity)		
+925,927.20**	(proceeds 2-year, 0% bonds)			–1,080,000	(maturity)
+2,850.40	Net	0		0	

*76,923.20 = 0.96154 × 80,000

**925,927.20 = 0.85734 × 1,080,000

The result is receiving *positive income today* in return for *no future obligation*, which is an *arbitrage opportunity*. The selling of the 2-year STRIPS would force the price down to 85.469 (the price at which the YTM = 8.167%), at which point the arbitrage opportunity would disappear.

Treasury Coupon Bonds and Treasury STRIPS

AIM 36.4: Discuss the components of a U.S. Treasury coupon bond, and compare and contrast the structure to Treasury STRIPS, including the difference between P-STRIPS and C-STRIPS.

Zero-coupon bonds issued by the Treasury are called **STRIPS** (separate trading of registered interest and principal securities). STRIPS are created by request when a coupon bond is presented to the Treasury. The bond is "stripped" into two components: principal and coupon (**P-STRIPS** and **C-STRIPS**, respectively).

The Treasury can also retire a STRIP by gathering the parts up to **reconstitute**, or remake, the coupon bond. C-STRIPS can be put with any bond to reconstitute, but P-STRIPS are identified with specific bonds—the original bond that it was stripped from. What this means is that the value of a P-STRIP comes from the underlying bond. If the underlying was cheap, the P-STRIP will be cheap. If the underlying was rich, the P-STRIP will also be rich.

STRIPS are of interest to investors because:

- Zero-coupon bonds can be easily used to create any type of cash flow stream and thus match asset cash flows with liability cash flows (e.g., to provide for college expenses, house-purchase down payment, or other liability funding). This mitigates reinvestment risk. (The concept of reinvestment risk will be discussed in later topics.)
- Zero-coupon bonds are more sensitive to interest rate changes than are coupon bonds. This could be an issue for asset-liability management or hedging purposes.

STRIPS do have some disadvantages, which include the following:

- They can be illiquid.
- Shorter-term C-STRIPS tend to trade rich.
- Longer-term C-STRIPS tend to trade cheap.
- P-STRIPS typically trade at fair value.
- Large institutions can potentially profit from STRIP mispricings relative to the underlying bonds. They can do this by either buying Treasuries and stripping them or reconstituting STRIPS. Because of the cost involved with stripping/reconstituting, investors generally pay a premium for zero-coupon bonds.

Constructing a Replicating Portfolio

AIM 36.5: Derive a replicating portfolio using multiple fixed-income securities in order to match the cash flows of a single given fixed income security.

Suppose a 2-year fixed income security exists with \$100 face value and a 10% coupon rate. The coupons are paid on a semiannual basis, and the security's YTM is assumed to be 4.5%. The present value of this bond, Bond 1, and its cash flows are calculated as follows:

$$PV_{B1} = \frac{5}{1.0225^1} + \frac{5}{1.0225^2} + \frac{5}{1.0225^3} + \frac{105}{1.0225^4} = \$110.41$$

or:

$$N = 2 \times 2 = 4;\ I/Y = 4.5/2 = 2.25\%;\ FV = 100;\ PMT = 10/2 = 5;\ CPT \Rightarrow PV = \$110.41$$

If this bond is determined to be trading "cheap," then a trader can conduct an arbitrage trade by purchasing the undervalued bond and shorting a replicating portfolio that mimics the bond's cash flows. To demonstrate the creation of a replicating portfolio, assume the following four fixed income securities exist in addition to the bond we are trying to replicate.

Bond	*Coupon*	*PV*	*FV*	*Time Horizon*
2	7%	\$101.22	\$100	6 months (0.5 years)
3	12%	\$107.25	\$100	12 months (1 year)
4	5%	\$100.72	\$100	18 months (1.5 years)
5	6%	\$102.84	\$100	24 months (2 years)

To create a replicating portfolio using multiple fixed-income securities, we must determine the face amounts of each bond to purchase, F_i, which match Bond 1 cash flows in each semiannual period.

$$\text{Bond 1 } CF_t = F_2 \times \frac{7\%}{2} + F_3 \times \frac{12\%}{2} + F_4 \times \frac{5\%}{2} + F_5 \times \frac{6\%}{2}$$

When doing this calculation by hand, it is easiest to start from the end—with the bond that matches Bond 1's time horizon. In this case, that security is Bond 5. Since the other bonds do not make payments in 24 months, they are not considered in this first step (i.e., their face amounts are multiplied by zero).

$$\$105 = F_2 \times 0 + F_3 \times 0 + F_4 \times 0 + F_5 \times \left(100 + \frac{6}{2}\right)\%$$

Solving this equation for F_5 yields the face amount percentage we need to purchase of Bond 5 (F_5 = 101.94). Since the coupon rate on Bond 5 is lower than that of Bond 1, it makes sense that we'll need to purchase more of Bond 5 (101.94%) than the \$100 face value of Bond 1. We can now use the value of F_5 to solve for F_4.

$$\$5 = F_2 \times 0 + F_3 \times 0 + F_4 \times \left(100 + \frac{5}{2}\right)\% + 101.94 \times \frac{6\%}{2}$$

The remaining unknowns (F_2 and F_3) are solved in a similar fashion. The replicating portfolio can now be purchased (or sold for the arbitrage trade) using the below face amount percentages. Notice, in the last two rows of the following table, how the total cash flows from these four bonds exactly matches the cash flows from Bond 1.

The cash flows from the replicating portfolio are computed by multiplying each bond's initial cash flows by face amount percentage. For example, regarding Bond 5, the 2-year cash flow is computed as \$103 × 1.0194 = \$105, and the 1-year cash flow is computed as \$3 × 1.0194 = \$3.0582.

	Coupon	*Face Amount*	*CF (t = 0.5)*	*CF (t = 1)*	*CF (t = 1.5)*	*CF (t = 2)*
Bond 2	7%	1.73%	1.7906			
Bond 3	12%	1.79%	0.1074	1.8974		
Bond 4	5%	1.89%	0.0473	0.0473	1.9373	
Bond 5	6%	101.94%	3.0582	3.0582	3.0582	105
Total CFs			5	5	5	105
Bond 1 CFs			5	5	5	105

Key Concepts

1. The cash flows of a coupon bond consist of periodic coupon payments and a par value payment at maturity.

2. The price of a bond consists of an ordinary annuity portion (the coupons) and a lump sum single cash flow (the par amount).

3. Bond prices can be generated from discount functions. Prices are calculated by summing the product of each cash flow and its applicable discount rate.

4. An arbitrage profit can be made if violations of the law of one price exist. By short selling a "rich" security and using the proceeds to purchase a similar "cheap" security, an investor can make a riskless profit with no investment.

5. Treasury STRIPS can be used to create specific fixed-income cash flow streams. P-STRIPS typically trade at fair value, while longer-term C-STRIPS tend to trade cheap, and shorter-term C-STRIPS tend to trade rich.

Concept Checkers

Use the following information for Questions 1 and 2.

Maturity	*Coupon*	*Price*
6 months	5.5%	101.3423
1 year	14.0%	102.1013
2 years	8.5%	99.8740

1. Which of the following is the closest to the discount factor for *d*(0.5)?
   A. 0.8923.
   B. 0.9304.
   C. 0.9525.
   D. 0.9863.

2. Which of the following is the closest to the discount factor for *d*(1.0)?
   A. 0.8897.
   B. 0.9394.
   C. 0.9525.
   D. 0.9746.

3. A Treasury bond is quoted at a price of 106-17+. The price of the bond as a percent of par is closest to:
   A. 106.1700%.
   B. 106.1750%.
   C. 106.5313%.
   D. 106.5469%.

4. P-STRIPS typically:
   A. trade rich.
   B. trade cheap.
   C. trade at fair value.
   D. Not enough information.

5. Which of the following statements about STRIPS is correct? STRIPS:
   I. have less interest rate sensitivity than coupon bonds.
   II. tend to be highly liquid.

   A. I only.
   B. II only.
   C. Both I and II.
   D. Neither I nor II.

Concept Checker Answers

1. **D** $101.3423 = 102.75\ d(0.5)$

   $d(0.5) = 0.9863$

2. **A** $102.1013 = 7\ d(0.5) + 107\ d(1)$

   $102.1013 = 7(0.9863) + 107\ d(1)$

   $95.1972 = 107\ d(1)$

   $d(1) = 0.8897$

3. **D** The price of the bond is $106\ {}^{17.5}\!/_{32}\%$ of par, or 106.5469%.

4. **C** P-STRIPS usually trade at fair value. This means that the cheapness or richness of the underlying bond passes on to the P-STRIP.

5. **D** STRIPS can be relatively illiquid and have more interest rate sensitivity than coupon bonds. Because of the cost to strip/reconstitute, only large institutional investors can potentially profit from doing so. STRIPS are often used with hedging strategies for asset-liability management such as matching maturity dates with a liability stream.

The following is a review of the Valuation and Risk Models principles designed to address the AIM statements set forth by GARP®. This topic is also covered in:

Bond Prices, Spot Rates, and Forward Rates

Topic 37

Exam Focus

Any bond can be partitioned into a series of periodic cash flows. If we compute the present value of each cash flow, viewed as STRIPS, and add them up, we arrive at the value of the bond. In other words, a bond is really a package of STRIPS. Using this framework enables us to relate the yield curve directly to the spot curve. The spot curve may then be manipulated to compute a forward curve that represents interest rates between future periods implied by the current spot curve. In either case, STRIPS or discount factors can be used to calculate prices.

Annual Compounding vs. Semiannual Compounding

AIM 37.1: Calculate and describe the impact of different compounding frequencies on a bond's value.

Most financial institutions pay and charge interest over much shorter periods than annually. For instance, if an account pays interest every six months, we say interest is compounded semiannually. Every three months represents quarterly compounding, and every month is monthly compounding.

Use the following formula to find the future value of a bond using different compounding methods:

$$FV_n = PV_0 \times \left[1 + \frac{r}{m}\right]^{m \times n}$$

where:
r = annual rate
m = number of compounding periods per year
n = number of years

Assume $100 was invested for four years earning 10% compounded semiannually. After four years the future value would be:

$$FV_n = 100 \times \left[1 + \frac{0.10}{2}\right]^{2 \times 4} = \$147.75$$

Assuming annual compounding the future value would be:

$$FV_n = 100 \times \left[1 + \frac{0.10}{1}\right]^{1\times4} = \$146.41$$

The additional $1.34 (= 147.75 – 146.41) is the extra interest earned from the compounding effect of interest on interest. Although the differences do not seem profound, the effects of compounding are magnified with larger values, greater number of compounding periods per year, and/or higher nominal interest rates.

Holding Period Return

AIM 37.2: Calculate holding period returns under different compounding assumptions.

We can rearrange the previous future value of a bond calculation and solve for the rate of return, *r* (i.e., the holding period return). The rate of return on a bond is as follows:

$$r = m\left[\left(\frac{FV_n}{PV_0}\right)^{\frac{1}{m\times n}} - 1\right]$$

Assume $100 was initially invested and grew to $147.75 after four years. Using semiannual compounding yields the following rate of return:

$$r = 2\left[\left(\frac{\$147.75}{\$100}\right)^{\frac{1}{2\times4}} - 1\right] = 10\%$$

Professor's Note: Recall from the Time Value of Money topic in Book 2 that this type of problem can be easily solved with a financial calculator. N = 4 × 2; FV = 147.75; PV = –100; CPT I/Y = 5% × 2 = 10%.

The Spot Rate Curve

AIM 37.3: Derive spot rates from discount factors.

A *t*-period **spot rate**, denoted as *z(t)*, is the yield to maturity on a zero-coupon bond that matures in *t*-years (assuming semiannual compounding). The **spot rate curve** is the graph of the relationship between spot rates and maturity. The spot rate curve can be derived from either a series of STRIPS prices, or the comparable discount factors.

Example: Computing a spot rate

The price of a 6-month $100 par value STRIP is 99.2556. **Calculate** the 6-month annualized spot rate.

Answer:

You can use the Texas Instruments BA II Plus® to solve this problem. Here are the keystrokes:

$$N = 1; PV = -99.2556; PMT = 0; FV = 100; CPT \rightarrow I / Y = 0.75\%$$

$$z(0.5) = 0.75\% \times 2 = 1.50\%$$

Recall from the previous topic that the *t*-period *discount factor* is the present value today of $1 to be received at the end of period *t*. For semiannual coupon bonds, the *t*-year discount factor is related to the *t*-year spot rate as follows:

$$z(t) = 2\left[\left(\frac{1}{d(t)}\right)^{1/2t} - 1\right]$$

Notice that the 6-month discount factor (0.992556) is just the price of the 6-month STRIP (99.2556) expressed in decimal form. This means that either spot rates or discount factors can be used to price coupon bonds.

Example: Computing spot rates from STRIP prices

Given the STRIPS prices in **Figure 1**, **compute** the discount factors and spot rates for maturities ranging from six months to three years, and **graph** the spot rate curve.

Figure 1: STRIPS Prices and Discount Factors

Maturity (Years)	*STRIPS Price*	*Discount Factor*
0.5	99.2556	0.992556
1.0	97.8842	0.978842
1.5	96.2990	0.962990
2.0	94.3299	0.943299
2.5	92.1205	0.921205
3.0	89.7961	0.897961

Answer:

~~Consider the calculations for the 2.5-year maturity. In this case:~~

$$N = 5; PV = -92.1205; PMT = 0; FV = 100; CPT \rightarrow I/Y = 1.655\%$$

$$z(2.5) = 1.655\% \times 2 = 3.31\% \text{ or } 2\left[\left(\frac{100}{92.1205}\right)^{\frac{1}{5}} - 1\right] = 3.31\%$$

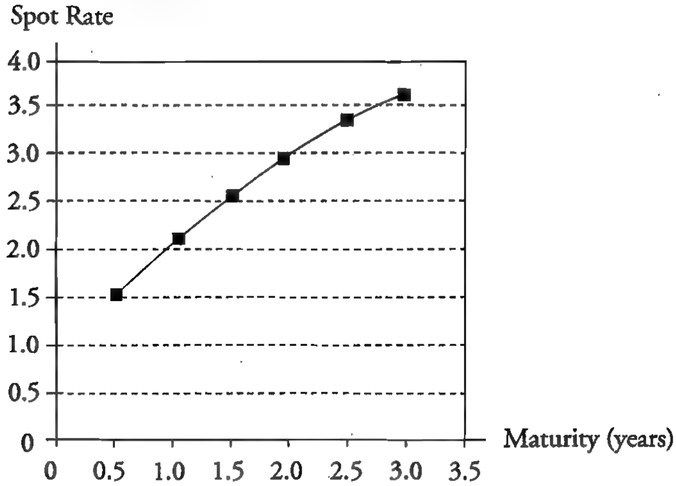
The spot rates for each maturity are shown in Figure 2.

Figure 2: Spot Rates

Maturity (Years)	*Spot Rate*
0.5	1.50%
1.0	2.15%
1.5	2.53%
2.0	2.94%
2.5	3.31%
3.0	3.62%

The resulting spot rate curve is shown in Figure 3.

Figure 3: Spot Rate Curve



FORWARD RATES

AIM 37.5: Define and interpret the forward rate, and compute forward rates given spot rates.

Forward rates are interest rates that span future periods. Given the spot rates as in Figures 2 and 3, it is possible to compute forward rates implied by that spot curve. The spot rates in Figures 2 and 3 are the appropriate rates that an investor should expect to realize for various periods for a risk-free investment starting today. Should the investor be concerned whether the investment is composed of a single instrument or a series of shorter investments rolled over consecutively? No, because if the risk is the same, the realized return must be the same, regardless of how the investment is packaged. This concept is at the core of forward rate analysis.

For example, suppose an investor is faced with the following two strategies, based on the spot curve in Figures 2 and 3:

1. Buy a 1-year STRIP yielding 2.15%.

2. Buy a 6-month (0.5-year) STRIP yielding 1.50% and then roll that into another 6-month STRIP in six months at the 6-month forward rate.

The investor will be indifferent about which investment to use if both offer the same return at the end of one year. The spot curve can be used to compute what the forward rate must be for an investor to be indifferent between the two strategies. This process is called **bootstrapping**.

Example: Computing a forward rate

Compute the 6-month forward rate in six months, given the following spot rates:

$z(0.5) = 1.50\%$

$z(1.0) = 2.15\%$

Answer:

In order for strategies 1 and 2 to realize the same return, the 6-month forward rate, f(1.0), on an investment that matures in one year must solve the following equation:

$$\left(1+\frac{0.0215}{2}\right)^2 = \left(1+\frac{0.0150}{2}\right)^1 \times \left(1+\frac{f(1.0)}{2}\right)^1$$

$$\Rightarrow f(1.0) = 0.028 = 2.80\%$$

Example: Computing a forward rate

Compute the 6-month forward rate in one year, given the following spot rates:

z(1.0) = 2.15%

z(1.5) = 2.53%

Answer:

The 6-month forward rate, f(1.5), on an investment that matures in 1.5 years must solve the following equation:

$$\left(1+\frac{0.0253}{2}\right)^3=\left(1+\frac{0.0215}{2}\right)^2\times\left(1+\frac{f(1.5)}{2}\right)^1$$

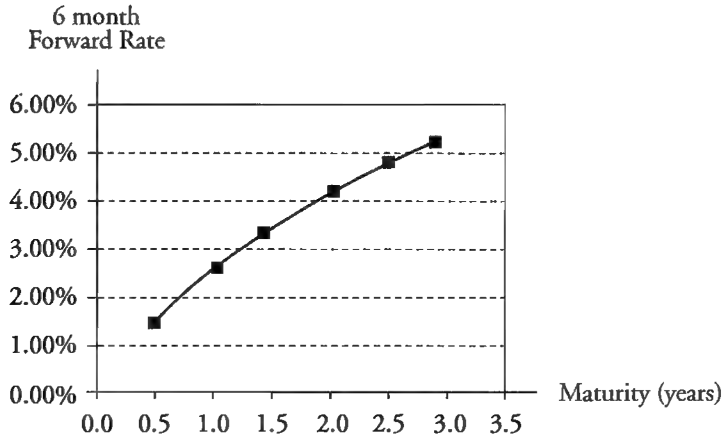
$$\Rightarrow f(1.5)=3.29\%$$

The remaining 6-month forward rates are shown in Figure 4, and the forward rate curve is shown in Figure 5.

Figure 4: Spot Rates and Forward Rates

Maturity (Years)	*Spot Rate*	*6-Month Forward Rate*
0.5	1.50%	1.50%
1.0	2.15%	2.80%
1.5	2.53%	3.29%
2.0	2.94%	4.18%
2.5	3.31%	4.80%
3.0	3.62%	5.18%

Figure 5: Forward Rate Curve



Pricing a Bond

AIM 37.4: Calculate the value of a bond using spot rates.

Any bond can be split into a series of periodic cash flows. Each cash flow in isolation can be considered a STRIP. If the present value of each cash flow is computed and summed, the resulting number should be equivalent to the bond's price. The appropriate discount rate for each cash flow is the spot rate. Discount factors can also be used because spot rates can be derived from discount factors.

Since spot rates and the implied forward rates are so closely related, it makes no difference which one is used to compute present values. A spot rate or a sequence of forward rates can be used to compute the present value. For example, a 1-year spot rate can be used to discount a cash flow taking place in one year, or a 6-month spot rate and the 6-month implied forward rate six months from now can be used. Both approaches will give the same present value, since they both span the same period.

Example: Calculating the price of a bond

Suppose a 1-year Treasury bond (T-bond) pays a 4% coupon semiannually. **Compute** its price using the discount factors, spot rates, and forward rates from Figure 6.

Figure 6: Discount Factors, Spot Rates, and Forward Rates

Maturity (Years)	*Discount Factor*	*Spot Rate*	*6-Month Forward Rate*
0.5	0.992556	1.50%	1.50%
1.0	0.978842	2.15%	2.80%
1.5	0.962990	2.53%	3.29%
2.0	0.943299	2.94%	4.18%
2.5	0.921205	3.31%	4.80%
3.0	0.897961	3.62%	5.18%

Answer:

Using discount factors:

$$\text{bond price} = (\$2 \times 0.992556) + (\$102 \times 0.978842) = \$101.83$$

Using spot rates:

$$\text{bond price} = \frac{\$2}{\left(1+\frac{0.0150}{2}\right)^1} + \frac{\$102}{\left(1+\frac{0.0215}{2}\right)^2} = \$101.83$$

Using forward rates:

$$\text{bond price} = \frac{\$2}{\left(1+\frac{0.0150}{2}\right)^1} + \frac{\$102}{\left(1+\frac{0.0150}{2}\right)^1 \times \left(1+\frac{0.0280}{2}\right)^1} = \$101.83$$

Effect of Maturity on Bond Prices and Returns

AIM 37.6: Discuss the impact of maturity on the price of a bond and the returns generated by bonds.

To analyze the effect of maturity on bond prices, we must compare coupon rates to corresponding forward rates over an arbitrary time period. In general, bond prices will tend to increase with maturity when coupon rates are above the relevant forward rates. The opposite holds when coupon rates are below the relevant forward rates (i.e., bond prices will tend to decrease with maturity in this scenario).

To analyze the effect of maturity on bond returns, assume two investors would like to invest over a 3-year time horizon. One investor invests in 6-month STRIPS and rolls them over for 3 years (i.e., when the first 6-month contract expires, he will invest in the next 6-month contract and so on for 3 years). The other investor just invests in a 3-year bond.

When short-term rates are above the forward rates utilized by bond prices, the investors who rolls over shorter-term investments will tend to outperform investors who invest in longer-term investments. The opposite holds when short-term rates are below the forward rates (i.e., the investor in long-term investments will outperform). If some short-term rates are lower than forward rates and some are higher, then a more detailed analysis will be required to determine which investor outperformed.

Yield Curve Calculations

AIM 37.7: Recognize the differences yield curve calculations yield when using P-STRIPS and C-STRIPS.

AIM 37.8: Define rich and cheap rates in the context of yield curves.

Analyzing P-STRIPS and C-STRIPS in the context of spot rates will yield similar conclusions to those discussed in the previous topic when analyzing the value of STRIPS over time. Short-term C-STRIPS will tend to trade rich (i.e., they tend to have lower spot rates). Long-term C-STRIPS will tend to trade cheap (i.e., they tend to have higher spot rates). Some P-STRIPS will trade rich if they exhibit high liquidity within a given maturity range since rates for on-the-run issues (i.e., newly issued T-bonds and T-notes) tend to be lower.

KEY CONCEPTS

1. Annual compounding means paying interest once a year, while semiannual compounding means paying interest once every six months.
2. A bond can be split into a package of STRIPS.
3. The yield curve for a given level of risk depicts the yield to maturity versus maturity.
4. Bootstrapping is the process of computing forward rates from spot rates.
5. A theoretical spot curve consisting of STRIPS can be computed by using discount factors and bootstrapping them.
6. Forward rates are interest rates corresponding to a future period implied by the spot curve.
7. All forward rates are computed using spot rates, and all spot rates can be computed using the appropriate forward rates.
8. In general, bond prices will increase with maturity when coupon rates are above relevant forward rates. A bond's return will depend on the duration of the investment and the relationship between spot and forward rates.

CONCEPT CHECKERS

Use the following data to answer Questions 1 and 2.

Maturity (Years)	*STRIPS Price*	*Spot Rate*	*Forward Rate*
0.5	98.7654	2.50%	2.50%
1.0	97.0662	3.00%	3.50%
1.5	95.2652	3.26%	3.78%
2.0	93.2775	?.??%	?.??%

1. The 6-month forward rate in 1.5 years (ending in year 2.0) is closest to:
   A. 4.04%.
   B. 4.11%.
   C. 4.26%.
   D. 4.57%.

2. The value of a 1.5-year, 6% semiannual coupon, $100 par value bond is closest to:
   A. $102.19.
   B. $103.42.
   C. $104.00.
   D. $105.66.

3. The 4-year spot rate is 8.36% and the 3-year spot rate is 8.75%. What is the 1-year forward rate three years from today (assuming these are annual rates)?
   A. 0.39%.
   B. 7.20%.
   C. 8.56%.
   D. 9.93%.

4. Given the interest rates, which of the following is closest to the price of a 4-year bond that has a par value of $1,000 and makes 10% coupon payments annually?

   - Current 1-year spot rate = 5.5%.
   - 1-year forward rate one year from today = 7.63%.
   - 1-year forward rate two years from today = 12.18%.
   - 1-year forward rate three years from today = 15.50%.

   A. $844.55.
   B. $995.89.
   C. $1,009.16.
   D. $1,085.62.

5. Given the following bonds and forward rates:

Maturity	*YTM*	*Coupon*	*Price*
1 year	4.5%	0%	95.694
2 years	7%	0%	87.344
3 years	9%	0%	77.218

- 1-year forward rate one year from today = 9.56%.
- 1-year forward rate two years from today = 10.77%.
- 2-year forward rate one year from today = 11.32%.

Which of the following statements about the forward rates, based on the bond prices, is true?

A. The 1-year forward rate one year from today is too low.
B. The 2-year forward rate one year from today is too high.
C. The 1-year forward rate two years from today is too low.
D. The forward rates and bond prices provide no opportunities for arbitrage.

Concept Checker Answers

1. **C** First compute the 2-year spot rate:

   N = 4; PV = –93.2775; PMT = 0; FV = 100; CPT I/Y = 1.755%

   z(0.5) = 1.755% × 2 = 3.51%

   Next compute the forward rate in 1.5 years ending in year 2.

   $$\left(1+\frac{0.0351}{2}\right)^4=\left(1+\frac{0.0326}{2}\right)^3\times\left(1+\frac{f(2.0)}{2}\right)^1$$

   $$\Rightarrow f(2.0)=4.26\%$$

2. **C** $$\text{bond price}=\frac{\$3}{\left(1+\frac{0.0250}{2}\right)^1}+\frac{\$3}{\left(1+\frac{0.0300}{2}\right)^2}+\frac{\$103}{\left(1+\frac{0.0326}{2}\right)^3}=\$104.00$$

3. **B** $$\frac{(1.0836)^4}{(1.0875)^3}-1=7.20\%$$

4. **C** The easiest way to find the bond value is to first calculate the appropriate spot rates to discount each cash flow.

   S_1 = 5.5%

   $S_2 = [(1.055)(1.0763)]^{1/2} - 1 = 6.56\%$

   $S_3 = [(1.055)(1.0763)(1.1218)]^{1/3} - 1 = 8.39\%$

   $S_4 = [(1.055)(1.0763)(1.1218)(1.155)]^{1/4} - 1 = 10.13\%$

   Then use the spot rates to discount each cash flow and take the sum of the discounted cash flows to find the value of the bond.

   $$\text{bond price }=\frac{\$100}{1.055}+\frac{\$100}{1.0656^2}+\frac{\$100}{1.0839^3}+\frac{\$1,100}{1.1013^4}=\$1,009.16$$

   Note that you could also do this in one step using the forward rates, but breaking the problem into two steps makes the math easier to do on your calculator.

5. **C** Given the bond spot rates on the zero-coupon bonds, the appropriate forward rates should be:

   - 1-year forward rate one year from today = $[(1 + 0.07)^2 / (1 + 0.045)] - 1 = 0.0956$, or 9.56%
   - 1-year forward rate two years from today = $[(1 + 0.09)^3 / (1 + 0.07)^2] - 1 = 0.1311$, or 13.11%
   - 2-year forward rate one year from today = $[(1 + 0.09)^3 / (1 + 0.045)] = 1.2393$. $1.2393^{0.5} - 1 = 0.1132 = 11.32\%$

   The 1-year forward rate two years from today is too low.

The following is a review of the Valuation and Risk Models principles designed to address the AIM statements set forth by GARP®. This topic is also covered in:

YIELD TO MATURITY

Topic 38

EXAM FOCUS

Bonds with coupons that are greater than market rates are said to trade at a premium, while bonds with coupon rates less than market rates are said to be trading at a discount. For coupon bonds, yield to maturity (YTM) is not a good measure of actual returns to maturity. When a bondholder receives coupon payments, the investor runs the risk that these cash flows will be reinvested at a rate of return that is lower than the original promised yield on the bond. This is known as reinvestment risk. For the exam, know how to calculate YTM given different compounding frequencies.

COMPUTING YTM

AIM 38.1: Define, interpret, and apply a bond's yield-to-maturity (YTM) to bond pricing.

AIM 38.2: Compute a bond's YTM given a bond structure and price.

The **yield to maturity**, or YTM, of a fixed-income security is equivalent to its internal rate of return. The YTM is the discount rate that equates the present value of all cash flows associated with the instrument to its price.

For a security that pays a series of known annual cash flows, the computation of yield uses the following relationship:

$$P=\frac{C_1}{(1+y)^1}+\frac{C_2}{(1+y)^2}+\frac{C_3}{(1+y)^3}+\ldots+\frac{C_N}{(1+y)^N}$$

where:
P = the price of the security
C_k = the annual cash flow in year k
N = term to maturity in years
y = the annual yield or YTM on the security

Example: Yield to maturity

Suppose a fixed-income instrument offers annual payments in the amount of $100 for ten years. The current value for this instrument is $700. **Compute** the YTM on this security.

Answer:

The YTM is the *y* that solves the following equation:

$$\$700 = \frac{\$100}{(1+y)^1} + \frac{\$100}{(1+y)^2} + \frac{\$100}{(1+y)^3} + \ldots + \frac{\$100}{(1+y)^{10}}$$

We can solve for YTM using a financial calculator:

N = 10; PMT = 100; PV = –700; CPT ⇒ I/Y = 7.07%

If cash flows occur more frequently than annually, the previous equation can be rewritten as:

$$P = \frac{C_1}{(1+y)^1} + \frac{C_2}{(1+y)^2} + \frac{C_3}{(1+y)^3} + \ldots + \frac{C_n}{(1+y)^n}$$

where:
n = N × m = the number of periods (years multiplied by payments per year)
C_k = the periodic cash flow in time period k
y = the periodic yield or periodic interest rate

Example: Periodic yield and YTM

Suppose now that the security in the previous example pays the $100 semiannually for five years. **Compute** the periodic yield and the YTM on this security.

Answer:

The periodic yield is the *y* that solves the following equation:

$$\$700 = \frac{\$100}{(1+y)^1} + \frac{\$100}{(1+y)^2} + \frac{\$100}{(1+y)^3} + \ldots + \frac{\$100}{(1+y)^{10}}$$

Using a financial calculator:

N = 10; PMT = 100; PV = –700; CPT ⇒ I/Y = 7.07%

Why is this the same value as in the previous example? Remember that this yield corresponds to a 6-month period. To compute the annual YTM, we must multiply the periodic yield by the number of periods per year, m = 2. This produces a YTM of 14.14%.

AIM 38.6: Describe the realized return on a bond and its relationship to YTM and reinvestment risk.

The yield to maturity can be viewed as the **realized return** on the bond assuming all cash flows are reinvested at the YTM.

Example: Realized return

Suppose a bond pays $50 every six months for five years and a final payment of $1,000 at maturity in five years. If the price is $900, **calculate** the realized return on the security. Assume all cash flows are reinvested at the YTM.

Answer:

The semiannual rate is the *y* that solves the following equation:

$$\$900 = \frac{\$50}{(1+y)^1} + \frac{\$50}{(1+y)^2} + \frac{\$50}{(1+y)^3} + \ldots + \frac{\$50 + \$1{,}000}{(1+y)^{10}}$$

Using a financial calculator, we arrive at a semiannual discount rate of 6.3835% and a YTM of 12.77%:

N = 10; PMT = 50; PV = –900; FV = 1,000; CPT ⇒ I/Y = 6.3835;
YTM = 6.3835 × 2 = 12.77%

The yield to maturity calculated above (2 × the semiannual discount rate) is referred to as a **bond equivalent yield** (BEY), and we will also refer to it as a semiannual YTM or semiannual-pay YTM. If you are given yields that are identified as BEY, you will know that you must divide by two to get the semiannual discount rate. With bonds that make annual coupon payments, we can calculate an **annual-pay yield to maturity**, which is simply the internal rate of return for the expected annual cash flows.

For zero-coupon Treasury bonds, the convention is to quote the yields as BEYs (semiannual-pay YTMs).

Example: Calculating YTM for zero-coupon bonds

A 5-year Treasury STRIP is priced at \$768. Calculate the semiannual-pay YTM and annual-pay YTM.

Answer:

The direct calculation method, based on the geometric mean, is:

$$\text{semiannual-pay YTM or BEY} = \left[\left(\frac{1{,}000}{768}\right)^{\frac{1}{10}} - 1\right] \times 2 = 5.35\%$$

$$\text{annual-pay YTM} = \left(\frac{1{,}000}{768}\right)^{\frac{1}{5}} - 1 = 5.42\%$$

The Limitations of Traditional Yield Measures

When a bondholder receives coupon payments, the investor runs the risk that these cash flows will be reinvested at a rate of return, or yield, that is lower than the promised yield on the bond. For example, if interest rates go down across the board, the reinvestment rate will also be lower. This is known as **reinvestment risk**. Reinvestment risk is a major threat to the bond's computed YTM, as it is assumed in such calculations that the coupon cash flows can be reinvested at a rate of return that's equal to the computed yield (i.e., if the computed yield is 8%, it is assumed the investor will be able to reinvest all coupons at 8%).

Reinvestment risk applies not only to coupons but also to the repayment of principal. Thus, it is present with bonds that can be prematurely retired, as well as with amortizing bonds where both principal and interest are received periodically over the life of the bond. Reinvestment risk becomes more of a problem with *longer term bonds* and with bonds that carry *larger coupons*. Reinvestment risk, therefore, is high for long-maturity, high-coupon bonds and is low for short-maturity, low-coupon bonds.

The realized yield on a bond is the actual compound return that was earned on the initial investment. It is usually computed at the end of the investment horizon. For a bond to have a **realized yield** equal to its YTM, all cash flows prior to maturity must be reinvested at the YTM, and the bond must be held until maturity. If the "average" reinvestment rate is below the YTM, the realized yield will be below the YTM. For this reason, it is often stated that: *The yield to maturity assumes cash flows will be reinvested at the YTM and assumes that the bond will be held until maturity.*

AIM 38.3: Calculate the price of an annuity and a perpetuity.

Calculating the Price of an Annuity

We can easily calculate the price of cash flows (annuities) if given the YTM and cash flows.

Example: Present value of an annuity

Suppose a fixed-income instrument offers annual payments in the amount of $100 for 10 years. The YTM for this instrument is 10%. **Compute** the price (PV) of this security.

Answer:

The price is the PV that solves the following equation:

$$PV = \frac{\$100}{(1+0.10)^1} + \frac{\$100}{(1+0.10)^2} + \frac{\$100}{(1+0.10)^3} + \ldots + \frac{\$100}{(1+0.10)^{10}}$$

Using a financial calculator the price equals $614.46:

N = 10; PMT = 100; I/Y = 10; CPT ⇒ PV = $614.46

Calculating the Price of a Perpetuity

The perpetuity formula is straightforward and does not require an iterative process:

$$\text{PV of a perpetuity} = \frac{C}{y}$$

where:
C = the cash flow that will occur every period into perpetuity
y = yield to maturity

Example: Price of perpetuity

Suppose we have a security paying $1,000 annually into perpetuity. The interest rate is 10%. **Calculate** the price of the perpetuity.

Answer:

We don't need a financial calculator to do this calculation. The price of the perpetuity is simply $10,000:

$$PV = \frac{\$1{,}000}{0.10} = \$10{,}000$$

Spot Rates and YTM

AIM 38.4: Establish the relationship between spot rates and YTM.

In the previous topic, we discussed the calculation of spot rates and examined how to value a bond given a spot rate curve. Pricing a bond using YTM is similar to using spot rates in that YTM is a blend of the given spot rates. Consider the following example.

Example: Spot rates and YTM

A bond with a \$100 par value pays a 5% coupon annually for 4 years. The spot rates corresponding to the payment dates are as follows:

Year 1: 4.0%
Year 2: 4.5%
Year 3: 5.0%
Year 4: 5.5%

Assume the price of the bond is \$98.47. **Show** the calculation of the price of the bond using spot rates and **determine** the YTM for the bond.

Answer:

The formula for the price of the bond using the spot rates is as follows:

$$P=\frac{5}{(1.04)}+\frac{5}{(1.045)^2}+\frac{5}{(1.05)^3}+\frac{105}{(1.055)^4}$$

$$\$98.47=4.81+4.58+4.32+84.76$$

Now compute the YTM:

$$\$98.47=\frac{5}{(1+\text{YTM})}+\frac{5}{(1+\text{YTM})^2}+\frac{5}{(1+\text{YTM})^3}+\frac{105}{(1+\text{YTM})^4}$$

$$\text{FV}=\$100;\ \text{PV}=-\$98.47;\ \text{PMT}=5;\ \text{N}=4;\ \text{CPT}\rightarrow \text{I}/\text{Y}=5.44\%$$

$$\text{YTM}=5.44\%$$

We see from this example that the YTM is closest to the 4-year spot rate. This is because the largest cash flow occurs at year 4 as the bond matures. If the spot curve is upward sloping, as in this example, the YTM will be less than the 4-year spot (i.e., the last spot rate). If the spot curve is flat, the YTM will be equal to the 4-year spot, and if the spot curve is downward sloping, the YTM will be greater than the 4-year spot.

The Relationship Between Par Value, Coupon Rate, and Price

AIM 38.5: Understand the relationship between coupon rate, YTM, and bond prices.

A bond's price reflects its relative value in the market based on several factors. Assume that a firm issues a bond at par, meaning that the market rate for the bond is precisely that of the coupon rate. Immediately after this bond is issued and before the market has time to adjust, the bond will trade at *par*. After the bond begins trading in the market, the same cannot be said. The price of the bond will reflect market conditions.

For example, suppose that after the bond was issued, market interest rates declined substantially. Investors in the bond would receive coupon rates substantially higher than what the market currently offers. Because of this, the price of the bond would adjust upward. This bond is a **premium bond**. If interest rates were to increase substantially after the bond was issued, investors would have to be compensated for the fact that the coupon rate of the bond is substantially lower than those offered currently in the market. The price would adjust downward as a consequence. The bond would be referred to as a **discount bond**.

Therefore:

- If coupon rate > YTM, the bond will sell for more than par value, or at a **premium**.
- If coupon rate < YTM, the bond will sell for less than par value, or at a **discount**.
- If coupon rate = YTM, the bond will sell for **par value**.

Over time, the price of premium bonds will gradually fall until they trade at par value at maturity. Similarly, the price of discount bonds will gradually rise to par value as maturity gets closer. This converging effect is known as **pull to par**.

Coupon Effect

If two bonds are identical in all respects except their coupon, ***the bond with the smaller coupon will be more sensitive to interest rate changes.*** That is, for any given change in yield, the smaller-coupon bond will experience a bigger percentage change in price than the larger-coupon bond. All else being equal:

- The lower the coupon rate, the greater the interest-rate risk.
- The higher the coupon rate, the lower the interest-rate risk.

Figure 1 summarizes the relationship between bond price sensitivity and coupon size. The bonds have equal maturities but different coupons. Assume semiannual coupons for both bonds.

Figure 1: Bond Price Reactions to Changes in Yield

	Price Change From Par ($1,000)	
Change in Interest Rates	*20-year, 8%*	*20-year, 12%*
–2%	+231.15	+171.59
–1%	+106.77	+80.23
0%	0	0
+1%	–92.01	–70.73
+2%	–171.59	–133.32

Coupon effect: For the same change in interest rates, the 20-year, 8% bond experiences a greater change in price than the 20-year, 12% bond. This suggests that bonds with similar maturities, but different coupon rates, can have different yield to maturities.

Key Concepts

1. Yield is an internal rate of return found by equating the present value of the cash flows to the current price of the security. An iterative process is used for the actual computation of yield. On a financial calculator, it can be found by inputting all other variables and solving for YTM.
2. When the bond is trading at par, the coupon rate is equal to the YTM. When the bond is trading below par, the coupon rate is less than the YTM, and is said to trade at a discount. When a bond is trading above par, the coupon rate is greater than the YTM, and the bond then trades at a premium.
3. The present value, or price, of a perpetuity can be found by dividing the coupon payment by the YTM.
4. Reinvestment risk occurs because in the yield-to-maturity calculation we assume that all periodic coupon payments can be reinvested at the YTM. In reality this is rarely the case. Reinvestment risk is higher for longer-term bonds and those that carry larger coupons.
5. When pricing a bond, YTM or spot rates can be used. The YTM will be a blend of the spot rates for the bond.

Concept Checkers

1. An annuity pays $10 every year for 100 years and currently costs $100. The YTM is closest to:
   A. 5%.
   B. 7%.
   C. 9%.
   D. 10%.

2. A $1,000 par bond carries a 7.75% semiannual coupon rate. Prevailing market rates are 8.25%. The price of the bond is:
   A. less than $1,000.
   B. greater than $1,000.
   C. $1,000.
   D. Not enough information to determine.

3. A $1,000 par bond carries a coupon rate of 10%, pays coupons semiannually, and has 13 years remaining to maturity. Market rates are currently 9.25%. The price of the bond is closest to:
   A. $586.60.
   B. $1,036.03.
   C. $1,055.41.
   D. $1,056.05.

4. Reinvestment risk would not occur if:
   A. interest rates shifted over the time period the bond is held.
   B. the bonds were callable.
   C. bonds are issued at par.
   D. only zero-coupon bonds are purchased.

5. An investment pays $50 annually into perpetuity and yields 6%. Which of the following is closest to the price?
   A. $120.
   B. $300.
   C. $530.
   D. $830.

Concept Checker Answers

1. D N = 100; PMT = 10; PV = –100; CPT ⇒ I/Y = 10%

2. A Since the coupon rate is less than the market interest rate, the bond is a discount bond and trades less than par.

3. D N = 26; PMT = 50; I/Y = 4.625; FV = 1,000; CPT ⇒ PV = $1,056.05

4. D Callable bonds have reinvestment risk because the principal can be prematurely retired. The higher the coupon, the higher the reinvestment risk, holding all else constant. A bond being issued at par has nothing to do with reinvestment risk.

5. D PV = C/I = $50 / 0.06 = $833.33

The following is a review of the Valuation and Risk Models principles designed to address the AIM statements set forth by GARP®. This topic is also covered in:

One-Factor Measures of Price Sensitivity

Topic 39

Exam Focus

The dollar value of a basis point, or DV01, measures how much the price of a bond changes from a one basis point change in yield. The DV01 is used when implementing fixed income hedging strategies. Another measure of price volatility or interest rate sensitivity is duration. Duration is an estimate of the percentage price change in the bond price from a change in the term structure. DV01 and duration ignore the nonlinear aspects of the price/yield relationship because they are linear approximations to a convex function. As the change in yield increases, these measures become progressively less accurate at predicting price changes. Convexity complements these measures by capturing the effects of the curvature of the price/yield relationship.

Measuring Price Sensitivity

AIM 39.1: Describe four ways in which measures of fixed income price sensitivity are used.

AIM 39.2: Describe an interest rate factor and name common examples of interest rate factors.

Measures of interest rate sensitivity allow investors to evaluate bond price changes as a result of interest rate changes. Being able to properly measure price sensitivity can be useful in the following situations:

1. Hedgers must understand how the bond being hedged as well as the hedging instrument used will respond to interest rate changes.
2. Investors need to determine the optimal investment to make in the event that expected changes in rates do in fact occur.
3. Portfolio managers would like to know the portfolio level of volatility for expected changes in rates.
4. Asset/liability managers need to match the interest rate sensitivity of their assets with the interest rate sensitivity of their liabilities.

In order to estimate bond price changes, we need to have some idea as to how interest rates will change going forward. Price changes are based on **interest rate factors**, which are random variables that influence individual interest rates along the yield curve. For this topic, we will be evaluating price sensitivity based on parallel shifts in the yield curve. This is a one-factor approach which assumes that a change in one rate (e.g., 20-year rate) will impact all other rates along the curve in a similar fashion.

Dollar Value of a Basis Point

AIM 39.3: Define and compute the DV01 of a fixed income security given a change in yield and the resulting change in price.

The price value of a basis point (PVBP) or the **dollar value of a basis point** (DV01) is the absolute change in bond price from a one basis point change in yield. Use the following relationship to compute the DV01:

$$\text{DV01} = |\text{price at YTM}_0 - \text{price at YTM}_1|$$

where:

$|\ |$ = the absolute value

YTM_0 = the initial yield to maturity

YTM_1 = the yield to maturity one basis point above or below YTM_0 $(\text{YTM}_1 = \text{YTM}_0 \pm 0.0001)$

Example: Computing DV01

Using semiannual discounting, **compute** the DV01 for a 20-year, 5% U.S. Treasury bond (T-bond) that is yielding 4.5%.

Answer:

Price at 4.49% = 106.6851% of par:

N = 20 × 2 = 40; I/Y = 4.49/2 = 2.245; PMT = 5/2 = 2.5; FV = 100;
CPT ⇒ PV = 106.6851

Price at 4.50% = 106.5484% of par:

N = 20 × 2 = 40; I/Y = 4.5/2 = 2.25; PMT = 5/2 = 2.5; FV = 100;
CPT ⇒ PV = 106.5484

Price at 4.51% = 106.4119% of par:

N = 20 × 2 = 40; I/Y = 4.51/2 = 2.255; PMT = 5/2 = 2.5; FV = 100;
CPT ⇒ PV = 106.4119

$\text{DV01}_{+\Delta y} = |106.5484 - 106.4119| = 0.1365$

$\text{DV01}_{-\Delta y} = |106.5484 - 106.6851| = 0.1367$

Notice that the two results are not exactly the same, due to the convexity of the price/yield relationship. As we will discuss later, the convexity relationships imply that a larger price increase occurs with a yield decrease than the price decrease associated with an identical yield increase. Clearly this is a limitation of DV01 since it assumes that the price/yield curve is linear. For large changes in interest rates, DV01 will do a poor job of estimating actual price sensitivity.

Price Changes vs. Percentage (Relative) Change

DV01 is an absolute dollar change from a one basis point change in yield. To convert this to a relative or percentage change we must divide it by the initial price of the bond:

$$\text{PPC} = \frac{\text{DV01}}{\text{price at YTM}_0}$$

where:
PPC = the percentage price change

Example: Absolute vs. relative price change

Compute the PPC for the 20-year, 5% U.S. T-bond in the previous example. Use the average DV01 and assume the yield is 4.5%.

Answer:

$$\text{PPC} = \frac{0.1366}{106.5484} = 0.13\%$$

DV01 Application to Hedging

AIM 39.4: Calculate the face amount of bonds required to hedge an option position given the DV01 of each.

Sensitivity measures like DV01 are commonly used to compute **hedge ratios**. Hedge ratios provide the relative sensitivity between the position to be hedged and the instrument used to hedge the position. For example, if the hedge ratio is 1, that means that the hedging instrument and the position have the same interest rate sensitivity.

The goal of a hedge is to produce a combined position (the initial position combined with the hedge position) that will not change in value for a small change in yield. This is expressed as:

dollar price change of position = dollar price change of hedging instrument

$$HR = \frac{\text{DV01 (per \$100 of initial position)}}{\text{DV01 (per \$100 of hedging instrument)}}$$

Example: Computing the amount of bonds needed to hedge

Suppose a 30-year semiannual coupon bond has a DV01 of 0.17195624, and a 15-year semiannual coupon bond will be used as the hedging instrument. The 15-year bond has a DV01 of 0.10458173. **Compute** the hedge ratio.

Answer:

$$HR = \frac{0.17195624}{0.10458173} = 1.644$$

For every $1 par value of the 30-year bond, short $1.644 of par of the 15-year bond.

Professor's Note: If you are given a yield beta, be sure to use it. The yield beta is the relationship between the yield of the bond and the implied yield of the hedging instrument. In the above example, if the yield beta is anything other than 1, you would multiple the yield beta by the DV01 of the initial position.

Example: Computing the amount of options needed to hedge

An investor buys a 15-year semiannual coupon bond yielding 4%. The bond has a market value of $100 and is currently trading at par. The bond's DV01 is 0.1119. The investor plans to hedge this bond position by selling call options. The DV01 of the call options when the rate is 4% is 0.1045. **Compute** the face value of the call options that is needed to hedge the 15-year bond position.

Answer:

Incorporating the face value of the bonds into the hedge ratio yields the following equation.

$$\text{option value} = \frac{\text{bond value} \times \text{DV01 (bond position)}}{\text{DV01 (option position)}}$$

$$\text{option value} = \frac{100 \times 0.1119}{0.1045} = \$107.08$$

So in order to hedge the interest rate sensitivity of this $100 face value bond, the investor must sell $107.08 face value of the call options.

Duration

AIM 39.5: Define, compute, and interpret the effective duration of a fixed income security given a change in yield and the resulting change in price.

Duration is the most widely used measure of bond price volatility. A bond's price volatility is a function of its coupon, maturity, and initial yield. Duration captures the impact of all three of these variables in a single measure. Just as important, a bond's duration and its price volatility are directly related (i.e., the longer the duration, the more price volatility there is in a bond). Of course, such a characteristic greatly facilitates the comparative evaluation of alternative bond investments.

The formula for effective duration, more commonly referred to as "duration," is:

$$\text{duration} = \frac{BV_{-\Delta y} - BV_{+\Delta y}}{2 \times BV_0 \times \Delta y}$$

where:
$BV_{-\Delta y}$ = estimated price if yield decreases by a given amount, Δy
$BV_{+\Delta y}$ = estimated price if yield increases by a given amount, Δy
BV_0 = initial observed bond price
Δy = change in required yield, in decimal form

Example: Computing duration

Suppose there is a 15-year, option-free noncallable bond with an annual coupon of 7% trading at par. **Compute** and **interpret** the bond's duration for a 50 basis point increase and decrease in yield.

Answer:

If interest rates rise by 50 basis points (0.50%), the estimated price of the bond falls to 95.586%.

N = 15; PMT = 7.00; FV = 100; I/Y = 7.50%; CPT ⇒ PV = –95.586

If interest rates fall by 50 basis points, the estimated price of the bond is 104.701%. Therefore, the duration of the bond is:

$$\text{duration} = \frac{104.701 - 95.586}{2(100)(0.005)} = 9.115$$

So, for a 100 basis point (1%) change in required yield, the expected price change is 9.115%. In other words, if the yield on this bond goes up by 1%, the price should fall by about 9.115%. If yield drops by 1%, the price of the bond should rise by approximately 9.115%.

Alternative Definitions of Duration

Macaulay duration is an estimate of a bond's interest rate sensitivity based on the time, in years, until promised cash flows will arrive. Since a 5-year zero-coupon bond has only one cash flow five years from today, its Macaulay duration is five. The change in value in response to a 1% change in yield for a 5-year zero-coupon bond is approximately 5%. A 5-year coupon bond has some cash flows that arrive earlier than five years from today (the coupons), so its Macaulay duration is less than five. The higher the coupon, the less the price sensitivity (duration) of a bond.

Macaulay duration is the earliest measure of duration, and because it was based on the time, duration is often stated as years. Because Macaulay duration is based on the expected cash flows for an option-free bond, it is not an appropriate estimate of the price sensitivity of bonds with embedded options.

Modified duration is derived from Macaulay duration and offers a slight improvement over Macaulay duration in that it takes the current YTM into account. Modified duration = Macaulay duration / (1 + periodic market yield). Like Macaulay duration, and for the same reasons, modified duration is not an appropriate measure of interest rate sensitivity for bonds with embedded options. For option-free bonds, however, effective duration (based on small changes in YTM) and modified duration will be very similar.

AIM 39.6: Contrast DV01 and effective duration as measures of price sensitivity.

As it turns out, we can use duration to calculate the DV01 as follows:

$$\text{duration} \times 0.0001 \times \text{bond value} = \text{DV01}$$

The following example demonstrates this calculation.

Example: Calculating the price value of a basis point

A bond has a market value of $100,000 and a duration of 9.42. What is the price value of a basis point (i.e., DV01)?

Answer:

Using the duration formula, the percentage change in the bond's price for a change in yield of 0.01% is 0.01% × 9.42 = 0.0942%. We can calculate 0.0942% of the original $100,000 portfolio value as 0.000942 × 100,000 = $94.20. If the bond's yield increases (decreases) by one basis point, the portfolio value will fall (rise) by $94.20. $94.20 is the (duration-based) DV01 for this bond. We could also directly calculate the DV01 for this bond by increasing the YTM by 0.01% and calculating the change in bond value. This would give us the DV01 for an increase in yield. This would be very close to our duration-based estimate because duration is a very good estimate of interest rate risk for small changes in yield.

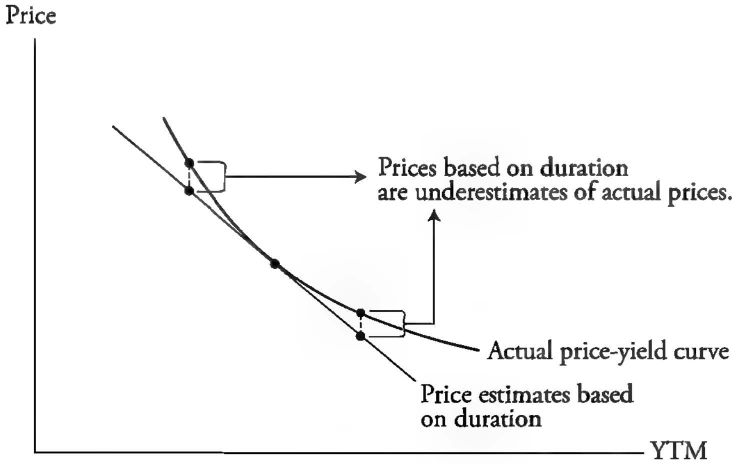
Both duration and DV01 convey a measure of price sensitivity for investors. However, the usefulness of these measures varies depending on the situation. If an investor simply wants to assess the riskiness of a security, then duration is the most intuitive measure since it allows investors to easily determine price change based on an estimated change in rates. If the investor is hedging an investment, as was illustrated in AIM 39.4, the use of DV01 is more appealing. This is because DV01 is able to accommodate differing face values of both the hedging instrument and the asset being hedged.

Convexity

AIM 39.7: Define, compute, and interpret the convexity of a fixed income security given a change in yield and the resulting change in price.

Duration is a good approximation of price changes for an option-free bond, but it is only good for relatively small changes in interest rates. Like DV01, duration is a linear estimate since it assumes that the price change will be the same regardless of whether interest rates go up or down. As rate changes grow larger, the curvature of the bond price/yield relationship becomes more important, meaning that a linear estimate of price changes will contain errors. Figure 1 illustrates why convexity is important and why estimates of price changes based solely on duration are inaccurate.

Figure 1: Duration-Based Price Estimates vs. Actual Bond Prices



While a precise calculation of a convexity involves the use of calculus (convexity is the second derivative of the price function with respect to yield), an approximate measure of convexity can be generated as follows:

$$\text{convexity} = \frac{BV_{-\Delta y} + BV_{+\Delta y} - 2 \times BV_0}{BV_0 \times \Delta y^2}$$

where:
$BV_{-\Delta y}$ = estimated price if yield decreases by a given amount, Δy
$BV_{+\Delta y}$ = estimated price if yield increases by a given amount, Δy
BV_0 = initial observed bond price
Δy = change in required yield, in decimal form

Given the magnitude of the change in yield, this equation essentially helps define the amount of convexity in the price/yield relationship.

Example: Computing convexity

Suppose there is a 15-year option-free noncallable bond with an annual coupon of 7% trading at par. If interest rates rise by 50 basis points (0.50%), the estimated price of the bond is 95.586%. If interest rates fall by 50 basis points, the estimated price of the bond is 104.701%. **Calculate** the convexity of this bond.

Answer:

$$\text{convexity} = \frac{104.701 + 95.586 - 2(100)}{(100)(0.005)^2} = 114.8$$

Unlike duration, the convexity of 114.8 cannot conveniently be converted into some measure of potential price volatility. Indeed, the convexity value means nothing in isolation, although a higher number does mean more price volatility than a lower number. This value can become very useful, however, when it is used to measure a bond's *convexity effect*, because it can be combined with a bond's duration to provide a more accurate estimate of potential price change.

Price Change Using Both Duration and Convexity

Now, by combining duration and convexity, a far more accurate estimate of the percentage change in the price of a bond can be obtained, especially for large swings in yield. That is, the amount of convexity embedded in a bond can be accounted for by adding the convexity effect to duration effect as follows:

percentage price change ≈ duration effect + convexity effect

$$= \left[-\text{duration} \times \Delta y \times 100\right] + \left[\left(\tfrac{1}{2}\right) \times \text{convexity} \times (\Delta y)^2 \times 100\right]$$

Example: Estimating price changes with the duration/convexity approach

Using the duration/convexity approach, **estimate** the effect of a 150 basis point increase and decrease on a 15-year, 7%, option-free bond currently trading at par. The bond has a duration of 9.115 and a convexity of 114.8.

Answer:

Using the duration/convexity approach:

$$\Delta V_{-}\% \approx [-9.115 \times -0.015 \times 100] + \left[\left(\tfrac{1}{2}\right) \times 114.8 \times (0.015)^2 \times 100\right]$$

$$= 13.6725\% + 1.2915\% = 14.9640\%$$

$$\Delta V_{+}\% \approx [-9.115 \times 0.015 \times 100] + \left[\left(\tfrac{1}{2}\right) \times 114.8 \times (0.015)^2 \times 100\right]$$

$$= -13.6725\% + 1.2915\% = -12.3810\%$$

Four important properties of convexity are:

1. As yields decrease (increase), the duration of a bond increases (decreases) at an increasing (decreasing) rate. Since convexity measures the rate of change of duration, it increases (decreases) as yields decrease (increase).

2. Holding yield constant, the lower the coupon, the higher the duration and the greater the convexity.

3. Holding *both* yield and duration constant, the lower the coupon, the lower the convexity. This rule suggests that convexity is also a measure of dispersion of cash flows.

4. Convexity increases at an increasing rate as duration increases.

Convexity's Role in Asset-Liability Management

With regard to portfolio management, duration alone is not a sufficient measure of interest rate exposure for bonds with significant convexity. When making investment decisions, managers often attempt to position their portfolios in a manner that will maximize profit from interest rate decreases, or minimize losses when rates are expected to rise. In this case, betting on the effects of convexity is essentially a play on volatility.

In the asset-liability arena, hedging is more effective if convexity, in addition to duration, is used to match the characteristics of the assets with the characteristics of the liability streams.

Portfolio Duration and Convexity

AIM 39.8: Calculate the effective duration and convexity of a portfolio of fixed income securities.

The portfolio duration, D_{Port}, of a bond portfolio is simply the value-weighted average of the durations of the bonds in the portfolio.

$$D_{Port} = \sum_{j=1}^{K} w_j \times D_j$$

where:
D_j = duration of bond j
w_j = market value of bond j divided by the market value of entire portfolio
K = the number of bonds in the portfolio

Like portfolio duration, portfolio convexity is calculated as the value-weighted average of the individual bond convexities within a portfolio.

Example: Computing portfolio duration

Using the portfolio as outlined in Figure 2, **calculate** the portfolio duration.

Figure 2: Portfolio Duration

Coupon	*Maturity (yrs.)*	*YTM*	*Price (% or par)*	*Par (millions)*	*Weights*	*D*
5.00%	5	4.00%	104.4912925	3	22.97%	4.41
6.00%	15	5.00%	110.4651463	4	32.37%	10.11
7.00%	30	5.50%	121.9169965	5	44.66%	13.97

Answer:

$$D_{Port} = (0.2297 \times 4.41) + (0.3237 \times 10.11) + (0.4466 \times 13.97) = 10.52$$

A significant problem with using portfolio duration as a measure of interest rate exposure is its implication that all the yields for every bond in the portfolio are perfectly correlated. This is a severely limiting assumption and should be of particular concern in global portfolios because it is unlikely that yields across national borders are perfectly correlated.

Determinants of Effective Duration

Effective duration is a function of the bond's maturity, yield, and credit rating. In general, the longer the term to maturity, all else equal, the greater the bond's duration and interest rate risk exposure. However, for deep-discount coupon bonds with maturities greater than 15 or 20 years, the duration actually falls slightly as maturity increases in order for its duration to equal the duration of a perpetuity.

The greater (lower) the yield to maturity, all else equal, the lower (greater) the bond's duration. In addition, the bond's yield is negatively related to its credit rating (i.e., the higher the credit rating, the lower the yield). Therefore, a credit rating upgrade (downgrade) will decrease (increase) the yield and increase (decrease) the bond's duration, all else equal.

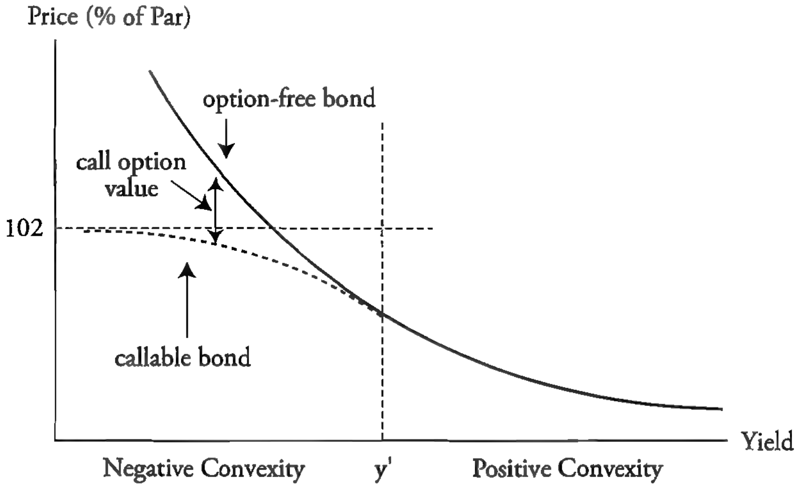
The Effect of Negative Convexity

AIM 39.9: Explain the effect negative convexity has on the hedging of fixed income securities.

With **callable debt**, the upside price appreciation in response to decreasing yields is limited (sometimes called price compression). Consider the case of a bond that is currently callable at 102. The fact that the issuer can call the bond at any time for \$1,020 per \$1,000 of face value puts an effective upper limit on the value of the bond. As Figure 3 illustrates, as yields fall and the price approaches \$1,020, the price-yield curve rises more slowly than that of an identical but noncallable bond. When the price begins to *rise at a decreasing rate* in response to further decreases in yield, the price-yield curve "bends over" to the left and exhibits **negative convexity**.

Thus, in Figure 3, so long as yields remain *below level y'*, callable bonds will exhibit *negative convexity*; however, at yields *above level y'*, those same callable bonds will exhibit *positive convexity*. In other words, at higher yields the value of the call options becomes very small, so that a callable bond will act very much like a noncallable bond. It is only at lower yields that the callable bond will exhibit negative convexity.

Figure 3: Price-Yield Function of a Callable vs. an Option-Free Bond



In terms of price sensitivity to interest rate changes, the slope of the price-yield curve at any particular yield tells the story. Note that as yields fall, the slope of the price-yield curve for the callable bond decreases, becoming almost zero (flat) at very low yields. This tells us how a call feature affects price sensitivity to changes in yield. At higher yields, the interest rate risk of a callable bond is very close or identical to that of a similar option-free bond. At lower yields, the price volatility of the callable bond will be much lower than that of an identical, but noncallable, bond.

Caution should be exercised whenever a bond that exhibits negative convexity is used to hedge a bond with positive convexity and vice versa. Any hedge of this nature is likely to become unstable when rates move away from y'.

The **yield to call** is used to calculate the yield on callable bonds that are selling at a premium to par. For bonds trading at a premium to par, the yield to call may be less than the yield to maturity. This can be the case when the call price is below the current market price.

The calculation of the yield to call is the same as the calculation of yield to maturity (shown in the previous topic) except that the *call price is substituted* for the par value in FV and the *number of semiannual periods until the call date is substituted* for periods to maturity, *N*. When a bond has a period of call protection, we calculate the yield to call over the period until the bond may first be called, and use the first call price in the calculation as FV.

Example: Computing the YTM and YTC

Consider a 20-year, 10% semiannual-pay bond with a full price of 112 that can be called in five years at 102. **Calculate** the YTM and yield to call.

Answer:

The YTM can be calculated as: N = 40; PV= –112; PMT= 5; FV= 100;
CPT → I/Y = 4.361% × 2 = 8.72% = YTM.

To compute the yield to call (YTC), we substitute the number of semiannual periods until the first call date (10) for *N*, and the first call price (102) for FV, as follows:

N = 10; PV= –112; PMT= 5; FV = 102;
CPT → I/Y = 3.71% and 2 × 3.71 = 7.42% = YTC

Key Concepts

1. The DV01, equivalently, the PVBP, is the absolute value of the price change of a bond from a one basis point change in yield. The DV01 is an adequate measure of price volatility or interest rate sensitivity since it corresponds to such a small change in yield.

2. The hedge ratio when hedging a bond with another bond is calculated as:

$$\text{HR} = \frac{\text{DV01 (initial position)}}{\text{DV01 (hedging instrument)}}$$

3. The formula for effective duration is:

$$\text{duration} = \frac{\text{BV}_{-\Delta y} - \text{BV}_{+\Delta y}}{2 \times \text{BV}_0 \times \Delta y}$$

4. Convexity is a measure of the degree of curvature or convexity in the price/yield relationship:

$$\text{convexity} = \frac{\text{BV}_{-\Delta y} + \text{BV}_{+\Delta y} - 2 \times \text{BV}_0}{\text{BV}_0 \times \Delta y^2}$$

5. Combining the standard convexity measure with a measure of duration provides more accurate estimates of bond price changes, particularly when the change in yield is relatively large.

percentage price change $\approx$ duration effect + convexity effect

$$= \left[-\text{duration} \times (\Delta y)\right] \times 100 + \left[\tfrac{1}{2} \times \text{convexity} \times (\Delta y)^2\right] \times 100$$

6. The use of convexity allows asset-liability management managers to better match the characteristics of their asset stream with that of their liabilities.

7. The portfolio duration of a bond portfolio is the value-weighted average of the durations of the bonds in the portfolio.

8. When hedging a bond that exhibits positive (negative) convexity with a bond that exhibits negative (positive) convexity, be cautious of interest rate movements. The bond's hedge may not be adequate after interest rates change.

CONCEPT CHECKERS

Use the following information to answer Questions 1 and 2.

An investor has a short position valued at $100 in a 10-year, 5% coupon, T-bond with a YTM of 7%. Assume discounting occurs on a semiannual basis.

1. Which of the following is closest to the dollar value of a basis point (DV01)?
   A. 0.065.
   B. 0.056.
   C. 0.047.
   D. 0.033.

2. Using a 20-year T-bond with a DV01 of 0.085 to hedge the interest rate risk in the 10-year bond mentioned above, which of the following actions should the investor take?
   A. Buy $130.75 of the hedging instrument.
   B. Sell $130.75 of the hedging instrument.
   C. Buy $76.50 of the hedging instrument.
   D. Sell $76.50 of the hedging instrument.

3. The duration of a portfolio can be computed as the sum of the value-weighted durations of the bonds in the portfolio. Which of the following is the most limiting assumption of this methodology?
   A. All the bonds in the portfolio must change by the same yield.
   B. The yields on all the bonds in the portfolio must be perfectly correlated.
   C. All the bonds in the portfolio must be in the same risk class or along the same yield curve.
   D. The portfolio must be equally weighted.

4. Estimate the percentage price change in bond price from a 25 basis point increase in yield on a bond with a duration of 7 and a convexity of 243.
   A. 1.67% decrease.
   B. 1.67% increase.
   C. 1.75% increase.
   D. 1.75% decrease.

5. An investor is estimating the interest rate risk of a 14% semiannual pay coupon bond with 6 years to maturity. The bond is currently trading at par. The effective duration and effective convexity of the bond for a 25 basis point increase and decrease in yield are closest to:

	Duration	Convexity
A.	3.970	23.20
B.	3.740	23.20
C.	3.970	20.80
D.	3.740	20.80

Concept Checker Answers

1. A For a 7% bond, N = 10 × 2 = 20; I/Y = 7/2 = 3.5%; PMT = 5/2 = 2.5; FV = 100; CPT → PV = –85.788

   For a 7.01% bond, N = 20; I/Y = 7.01/2 = 3.505%; PMT = 2.5; FV = 100; CPT → PV = –85.723

   For a 6.99% bond, N = 20; I/Y = 6.99/2 = 3.495%; PMT = 2.5; FV = 100; CPT → PV = –85.852

   DV01+Δy = | 85.788 – 85.723 | = 0.065

   DV01-Δy = | 85.788 – 85.852 | = 0.064

2. C The hedge ratio is 0.065 / 0.085 = 0.765. Since the investor has a short position in the bond, this means the investor needs to buy $0.765 of par value of the hedging instrument for every $1 of par value for the 10-year bond.

3. B A significant problem with using portfolio duration is that it assumes all yields for every bond in the portfolio are perfectly correlated. However, it is unlikely that yields across national borders are perfectly correlated.

4. A $\Delta V_{+}\% \approx [-7 \times 0.0025 \times 100] + \left[\left(\frac{1}{2}\right) \times 243 \times (0.0025)^2 \times 100\right] = -1.67\%$

5. C N = 12; PMT = 7; FV = 100; I/Y = 13.75/2 = 6.875%; CPT → PV = 100.999

   N = 12; PMT = 7; FV = 100; I/Y = 14.25/2 = 7.125%; CPT → PV = 99.014

   Δy = 0.0025

   $$\text{Duration} = \frac{100.999 - 99.014}{2(100)0.0025} = 3.970$$

   $$\text{Convexity} = \frac{100.999 + 99.014 - 200}{(100)(0.0025)^2} = 20.8$$

The following is a review of the Valuation and Risk Models principles designed to address the AIM statements set forth by GARP®. This topic is also covered in:

BINOMIAL TREES

Topic 40

EXAM FOCUS

This topic introduces the binomial model for valuing options on stock and serves as an introduction to the Black-Scholes-Merton model you'll encounter in the next topic. For the exam, you should be able to calculate the value of a European or American option using a one-step or two-step binomial model. This will require you to know the formulas for the sizes of upward and downward movements as well as the risk-neutral probabilities in both up and down states.

A ONE-STEP BINOMIAL MODEL

AIM 40.1: Calculate the value of a European call or put option using the one-step and two-step binomial model.

A **one-step binomial model** is best described within a two-state world where the price of a stock will either go up once or down once, and the change will occur one step ahead at the end of the holding period.

The Replicating Portfolio

The replicating portfolio is the key to understanding how to value options. In general, the replicating portfolio is a concept that holds that the outlay for a bankruptcy-free stock position should be the same as the outlay for a long call position with the same payoff.

To see how this works, let's first define some terms. Then we'll work through a calculation:

- P = the stock's current price.
- X = the call option's exercise price.
- t = the time to option expiration.
- i = the risk-free interest rate.
- S_U = the stock value in "up" state.
- S_D = the stock value in "down" state.
- c = the value of the call option today.

Example: Creating a replicating portfolio

Calculate the value of the call option where:

P = \$100
X = \$125
t = 1 year
i = 8%
S_U = \$200
S_D = \$50

The one-period binomial trees for the stock and the call option are shown in Figure 1.

Figure 1: One-Period Binomial Trees

Stock:	**Call Option:**
P = \$100 → S_U = \$200	c = ?? → c_U = max (\$0, \$200 – \$125) = \$75
P = \$100 → S_D = \$50	c = ?? → c_D = max (\$0, \$50 – \$125) = \$0

Answer:

The process used to establish a replicating portfolio can be broken down into four steps:

Step 1: Construct the bankruptcy-free portfolio. As shown in Figure 1, the stock's minimum value one year from today is \$50, which means we should borrow the present value of \$50: \$50/(1.08) = \$46.30. With the borrowed money and \$53.70 of our own money, purchase one share of stock for \$100. This stock-plus-borrowing combination is the bankruptcy-free portfolio, since:

- If the stock goes up to \$200, you will be able to repay your \$50 loan. Your net return (excluding out-of-pocket costs) will be \$200 – \$50 = \$150.
- If the stock goes down to \$50, you will still be able to repay your \$50 loan. Your net return will be \$50 – \$50 = \$0.

Current net cash outlay = \$53.70. Stock value net of loan = \$0 or \$150.

Step 2: Replicate the future returns. To replicate the stock-plus-borrowing transaction (i.e., the bankruptcy-free portfolio), we want our net cash returns from our option position (excluding out-of-pocket expenses) to equal \$0 or \$150 at the end of the holding period. Since the profit diagram for one call option indicated a net return of \$0 or \$75 at the end of the holding period, purchasing *two* call options will provide the same end-of-period return as the bankruptcy-free portfolio.

Step 3: ***Align the dollar cost of the option and the portfolio.*** If the dollar outlay for the bankruptcy-free portfolio is $53.70, the dollar cost of the two option contracts should also be $53.70, since they have the same risk and return. ***Arbitrage forces this condition.***

Step 4: ***Value the option.*** Since arbitrage forces the outlay of the bankruptcy-free portfolio to be the same as the outlay for two options, each option must cost half of the outlay for the bankruptcy-free portfolio. If two contracts cost $53.70, then c = $53.70/2 = $26.85. *Note*: To solve for the call price, all we needed were six data items: P, X, i, t, S_U and S_D.

Using the Hedge Ratio to Develop the Replicating Portfolio

The value of the option can also be solved by creating a **perfect hedge**.

- **Hedging** is the elimination of price variation through the short sale of an asset exhibiting the same price volatility as the asset to be hedged. A perfect hedge creates a riskless position.
- The **hedge ratio** indicates the number of asset units needed to completely eliminate the price volatility of one call option.

In the previous example we could have created a perfect hedge had we sold one share of stock short at $100 and purchased two call options. No matter which way the stock price moves, the hedged portfolio will be worth $50:

- If the stock price *falls* to $50, the two options will have zero value, so the net asset position is the gain on the short sale: $100 – $50 = $50.
- If the stock price *rises* to $200, the two calls will have a combined value of $150, leaving a net asset value of $50 after considering a $100 loss ($100 – $200) from the short position in the stock.

Since the terminal value of this strategy (short one share and long two calls) always nets $50, the present value of the strategy is $50/1.08 = $46.30. Therefore, the value of one call option must be $26.85 ($100 – 2c = $46.30, c = $26.85).

The hedge ratio tells us how many units of the stock are to be shorted per long call option to make the hedge work. In the single-period model, the hedge ratio may be calculated as follows:

$$HR = \frac{c_U - c_D}{S_U - S_D} = \frac{\$75 - \$0}{\$200 - \$50} = 0.5$$

A hedge ratio of 0.5 says that one option contract is needed for each half-share of stock. The reciprocal of the hedge ratio is equivalent to the number of option contracts to buy per share of stock that was sold short.

Synthetic Call Replication

A combination of the hedge ratio, the stock price, and the present value of the borrowings can be used to price the call option:

call price = hedge ratio × [stock price – PV(borrowing)]

Using the data from the previous example, the call price is:

call price = 0.5 × ($100 – $46.30) = $26.85

The hedge ratio is also known as the option **delta**, which is a key measure of option sensitivity, as you will see in Topic 42 when we discuss the Greek letters for option pricing.

Risk-Neutral Valuation

The one-step binomial model can also be expressed in terms of probabilities and call prices. The sizes of the upward and downward movements are defined as functions of the volatility and the length of the "steps" in the binomial model:

$$U = \text{size of the up-move factor} = e^{\sigma\sqrt{t}}$$

$$D = \text{size of the down-move factor} = e^{-\sigma\sqrt{t}} = \frac{1}{e^{\sigma\sqrt{t}}} = \frac{1}{U}$$

where:
σ = annual volatility of the underlying asset's returns
t = the length of the step in the binomial model

The risk-neutral probabilities of upward and downward movements are then calculated as follows:

$$\pi_u = \text{probability of an up move} = \frac{e^{rt} - D}{U - D}$$

$$\pi_d = \text{probability of a down move} = 1 - \pi_u$$

where:
r = continuously compounded annual risk-free rate

Professor's Note: These two probabilities are not the actual probability of an up or down move. They are risk-neutral probabilities that would exist if investors were risk-neutral.

We can calculate the value of an option on the stock by:

- Calculating the payoff of the option at maturity in both the up-move and down-move states.
- Calculating the expected value of the option in one year as the probability-weighted average of the payoffs in each state.
- Discounting the expected value back to today at the risk-free rate.

Example: Risk neutral approach to option valuation

The current price of Downhill Ski Equipment, Inc., is \$20. The annual standard deviation is 14%. The continuously compounded risk-free rate is 4% per year. Assume Downhill pays no dividends. Compute the value of a 1-year European call option with a strike price of \$20 using a one-period binomial model.

Answer:

The up-move and down-move factors are:

$$U = e^{0.14 \times \sqrt{1}} = 1.15$$

$$D = \frac{1}{1.15} = 0.87$$

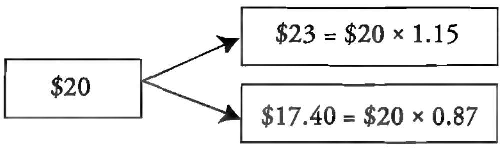
The risk-neutral probabilities of an up move and down move are:

$$\pi_u = \frac{e^{0.04 \times 1} - D}{U - D} = \frac{1.0408 - 0.87}{1.15 - 0.87} = 0.61$$

$$\pi_d = 1 - 0.61 = 0.39$$

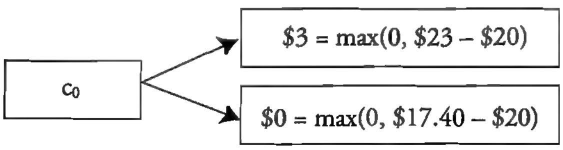
The binomial tree for the stock is shown in Figure 2:

Figure 2: Binomial Tree—Stock



The binomial tree for the option is shown in Figure 3:

Figure 3: Binomial Tree—Option



Notice that the call option is in-the-money in the "up" state, so its value is \$3. It is out-of-the-money in the "down" state, so its value is zero.

The expected value of the option in one year is:

$$c_U \times \pi_U + c_D \times \pi_D \text{ or } (\$3 \times 0.61) + (\$0 \times 0.39) = \$1.83$$

The present value of the option's expected value is:

$$c_0 = \frac{\$1.83}{e^{0.04 \times 1}} = \frac{\$1.83}{1.0408} = \$1.76$$

Example: Put option valuation using put-call parity

The current price of Downhill Ski Equipment, Inc., is \$20, the risk-free rate is 4% per year, and the price of a 1-year call option with a strike price of \$20 is \$1.76. **Compute** the value of a 1-year European put option on Downhill Ski Equipment with a strike price of \$20.

Answer:

$$\text{put} = \text{call} - \text{stock} + Xe^{-rT}$$

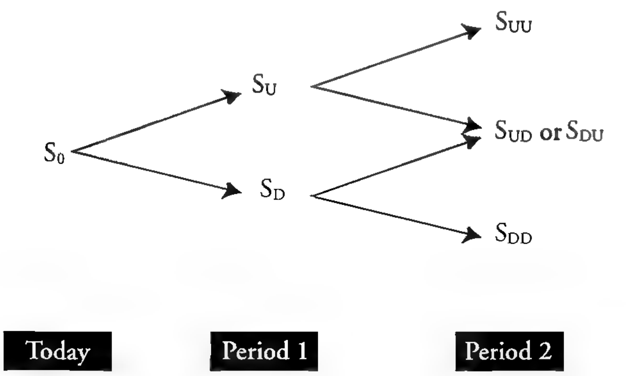
Using the information provided, we have:

$$\text{put} = \$1.76 - \$20 + [\$20 \times e^{-(0.04) \times (1)}] = \$0.98$$

Two-Step Binomial Model

In the two-period and multi-period models, the *tree* is expanded to provide for a greater number of potential outcomes. The stock price tree for the two-period model is shown in Figure 4.

Figure 4: Two-Step Binomial Model Stock Price Tree



Example: Option valuation with a two-step binomial model

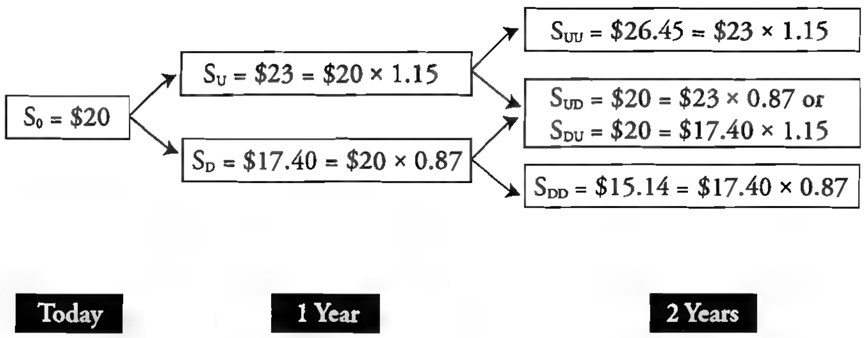
Let's continue with the Downhill Ski Equipment example. The current price of Downhill Ski Equipment, Inc., is $20. The annual standard deviation is 14%. The risk-free rate is 4% per year.

Assume Downhill pays no dividends. Using information from the previous example, **compute** the values of a 2-year European call and a 2-year European put option with strike prices of $20.

Answer:

First **compute** the theoretical value of the stock in each period using the up and down stock price movements from the preceding examples, as shown in Figure 5:

Figure 5: Theoretical Stock Value



From this, the values of the call option in each of the possible outcomes can be determined. Notice that the only time that the option is in-the-money is when two upward price movements lead to an ending price of $26.45 and a call value of $6.45. The expected value of the option at the end of year 2 is the value of the option in each state multiplied by the probability of that state occurring.

$$\begin{aligned}\text{expected call value in 2 years} &= (0.61 \times 0.61 \times \$6.45) + (0.61 \times 0.39 \times \$0)\\ &\quad + (0.39 \times 0.61 \times \$0) + (0.39 \times 0.39 \times \$0)\\ &= (0.3721 \times \$6.45) = \$2.40\end{aligned}$$

The value of the option today is the expected value in two years discounted at the risk-free rate of 4%:

$$\text{call option value} = \frac{\$2.40}{e^{(0.04)\times(2)}} = \$2.21$$

$$\text{put} = \text{call} - \text{stock} + Xe^{-rT} = \$2.21 - \$20 + \$20e^{-(0.04)(2)} = \$0.67$$

Assessing Volatility

AIM 40.3: Discuss how volatility is captured in the binomial model.

Notice from the previous examples that a high standard deviation will result in a large difference between the stock price in an up state, S_U, and the stock price in a down state, S_D. If the standard deviation were zero, the binomial tree would collapse into a straight line and S_U would equal S_D. Obviously, the higher the standard deviation, the greater the dispersion between stock prices in up and down states. Therefore volatility, as measured here by standard deviation, can be captured by evaluating stock prices at each time period considered in the tree.

Modifying the Binomial Model

AIM 40.5: Explain how the binomial model can be altered to price options on: stocks with dividends, stock indices, currencies, and futures.

The binomial option pricing model can be altered to value a stock that pays a continuous dividend yield, *q*. Since the total return in a risk-neutral setting is the risk-free rate, *r*, and dividends provide a positive yield, capital gains must be equal to r – q. The risk-neutral probabilities of upward and downward movements incorporate a dividend yield as follows:

$$\pi_u = \frac{e^{(r-q)t} - D}{U - D}$$

$$\pi_d = 1 - \pi_u$$

The equations for the size of the up-move and down-move factors will be the same. Options on stock indices are valued in a similar fashion to stocks with dividends, because it is assumed that stocks underlying the index pay a dividend yield equal to *q*.

For options on currencies, it is assumed that a foreign currency asset provides a return equal to the foreign risk-free rate of interest, r_{FC}. As a result, the upward probability in the binomial model is altered by replacing e^{rt} with $e^{(r_{DC}-r_{FC})t}$ such that:

$$\pi_u = \frac{e^{(r_{DC}-r_{FC})t} - D}{U - D}$$

The binomial model can also incorporate the unique characteristics of options on futures. Since futures contracts are costless to enter into, they are considered, in a risk-neutral setting, to be zero growth instruments. To account for this characteristic, e^{rt} is simply replaced with a 1 so that:

$$\pi_u = \frac{1 - D}{U - D}$$

American Options

AIM 40.2: Calculate the value of an American call or put option using a two-step binomial model.

Valuing American options with a binomial model requires the consideration of the ability of the holder to exercise early. In the case of a two-step model, that means determining whether early exercise is optimal at the end of the first period. If the payoff from early exercise (the intrinsic value of the option) is greater than the option's value (the present value of the expected payoff at the end of the second period), then it is optimal to exercise early.

Example: American put option valuation

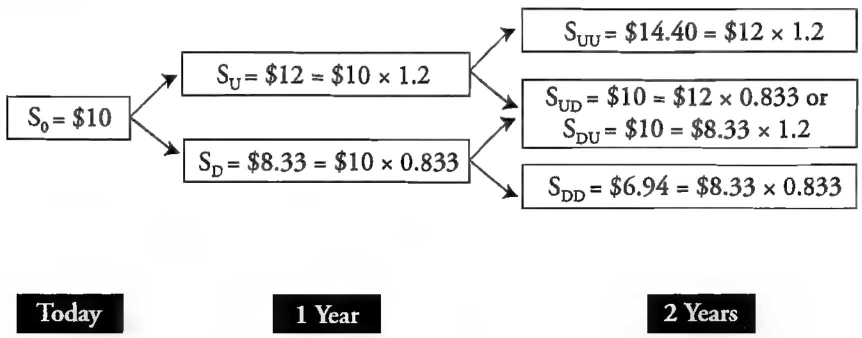
The current price of Uphill Mountaineering is $10. The up-move factor is 1.20, and the down-move factor is 0.833. The probability of an up move is 0.51, and the probability of a down move is 0.49. The risk-free rate is 2%. **Compute** the value of a 2-year American put option with strike price of $12.

Professor's Note: The calculation of the risk-neutral probabilities depends on the length of the time step. So, for a 2-year option with two time steps, the change in t *is 1 year. For example, the probability of an up move in the information above is calculated as:* $(e^{0.02 \times 1} - 0.833) / (1.2 - 0.833) = 0.51$.

Answer:

The stock price tree is shown in Figure 6.

Figure 6: Stock Price Tree



The $12 put option is in-the-money when the stock price finishes at $10 or at $6.94; the option is worth $2.00 ($12 – $10) or $5.06 ($12 – $6.94). It is out-of-the-money at $14.40. The year 1 value of the expected payoff on the option in year 2, given that the year 1 move is an up move, is:

$$\frac{(\$0.00 \times 0.51) + (\$2.00 \times 0.49)}{e^{(0.02)(1)}} = \$0.96$$

The payoff from early exercise at the year 1 up node is:

max ($12 – $12, 0), since the option is at-the-money

Early exercise is not optimal in this case because the option is worth more unexercised ($0.96), than if exercised ($0).

At the down node at the end of year 1, the value of the expected option payoff in year 2 is:

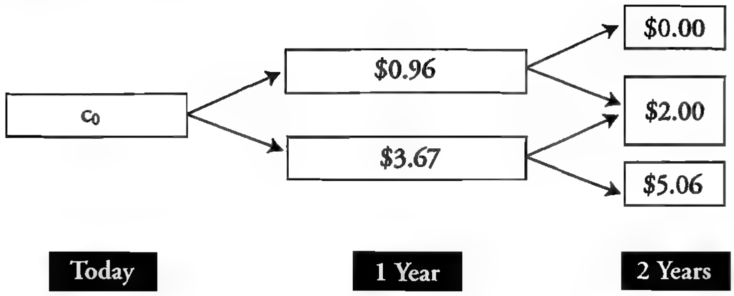
$$\frac{(\$2.00 \times 0.51) + (\$5.06 \times 0.49)}{e^{(0.02)(1)}} = \$3.43$$

The payoff from early exercise at the down node at the end of the first year is:

max ($12 – $8.33) = $3.67

In this case, early exercise is optimal because the option is worth more if exercised ($3.67) than if not exercised ($3.43). The option tree is shown in Figure 7.

Figure 7: Option Tree



The value of the option today is calculated as:

$$\frac{(\$0.96 \times 0.51) + (\$3.67 \times 0.49)}{e^{(0.02)(1)}} = \$2.24$$

Note that $3.67 appears in the bottom node of year 1 since the early exercise value ($3.67) exceeds the unexercised value ($3.43).

Professor's Note: When evaluating American options, you need to assess early exercise at each node in the tree. This includes the initial node (node 0). If the option price today (calculated via the binomial model) is less than the value of early exercise today, then the option should be exercised early. In the previous example, if the value of the option today was worth less than $2, the option would be exercised today since the put option is currently equal to $2.

Increasing the Number of Time Periods

AIM 40.4: Discuss how the binomial model value converges as time periods are added.

If we shorten the length of the intervals in a binomial model, there are more intervals over the same time period, more branches to consider, and more possible ending values. If we continue to shrink the length of intervals in the model until they are what mathematicians call "arbitrarily small," we approach continuous time as the limiting case of the binomial model. The model for option valuation in the next topic (the Black-Scholes-Merton model) is a continuous time model. The binomial model "converges" to this continuous time model as we make the time periods arbitrarily small.

KEY CONCEPTS

1. The value of a European option can be calculated using a binomial tree, as the probability-weighted expected value of the option at maturity discounted at the risk-free rate.

2. Given the volatility of the underlying stock and the length of the steps in the binomial tree, the size of the up- and down-move factors are calculated as:

   $$U = \text{size of the up-move factor} = e^{\sigma\sqrt{t}}$$

   $$D = \text{size of the down-move factor} = \frac{1}{U}$$

3. The risk-neutral probabilities of up and down moves are calculated as:

   $$\pi_u = \text{probability of an up move} = \frac{e^{rt} - D}{U - D}$$

   $$\pi_d = \text{probability of a down move} = 1 - \pi_u$$

   where:
   r = annual continuously compounded risk-free rate

4. The value of the comparable European put option can be calculated using put-call parity, which is put = call – stock + Xe^{-rT}.

5. The value of an American option reflects the early exercise features. An American option will be exercised at the end of the first period if the intrinsic value is greater than the discounted value of the expected option payoff at the end of the second period.

6. As the period covered by a binomial model is divided into arbitrarily small, discrete time periods, the model results converge to those of the continuous-time model.

Concept Checkers

1. The stock price is currently \$80. The stock price annual up-move factor is 1.15. The risk-free rate is 3.9%. The value of a 2-year European call option with an exercise price of \$62 using a two-step binomial model is closest to:
   A. \$0.00.
   B. \$18.00.
   C. \$23.07.
   D. \$24.92.

2. The stock price is currently \$80. The stock price will move up by 15% each year. The risk-free rate is 3.9%. The value of a 2-year European put option with an exercise price of \$62 using a two-step binomial model is closest to:
   A. \$0.42.
   B. \$16.89.
   C. \$18.65.
   D. \$21.05.

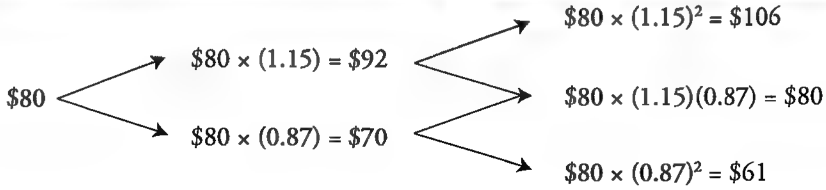
3. JTE Corporation is a nondividend-paying stock that is currently priced at \$49. An analyst has determined that the annual standard deviation of returns on JTE stock is 8% and that the annual risk-free interest rate on a continuously compounded basis is 5.5%. Calculate the value of a 6-month American call option on JTE stock with a strike price of \$50 using a two-period binomial model.
   A. \$0.32.
   B. \$0.65.
   C. \$1.31.
   D. \$2.97.

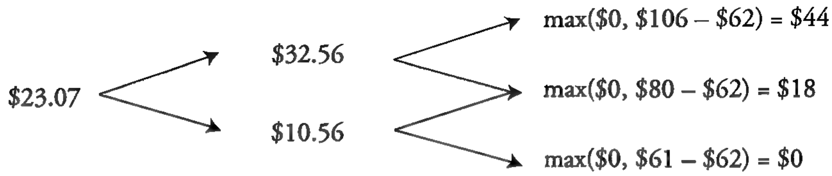
4. A 1-year American put option with an exercise price of \$50 will be worth either \$8.00 at maturity with a probability of 0.45 or \$0 with a probability of 0.55. The current stock price is \$45. The risk-free rate is 3%. The optimal strategy is to:
   A. exercise the option because the payoff from exercise exceeds the present value of the expected future payoff.
   B. not exercise the option because the payoff from exercise is less than the discounted present value of the future payoff.
   C. exercise the option because it is currently in-the-money.
   D. not exercise the option because it is currently out-of-the-money.

5. Suppose a 1-year European call option exists on XYZ stock. The current continuously compounded risk-free rate is 3%, and XYZ pays a continuous dividend yield of 2%. Assume an annual standard deviation of 3%. The risk-neutral probability of an up-move for the XYZ call option is:
   A. 0.67.
   B. 0.97.
   C. 1.00.
   D. 1.03.

Concept Checker Answers

1. C





$$U = 1.15$$

$$D = \frac{1}{1.15} = 0.8696$$

$$\pi_U = \frac{\left(e^{0.039}\right) - (0.87)}{1.15 - 0.87} = 0.61$$

$$\pi_D = 1 - 0.61 = 0.39$$

$$\pi_{UU} = 0.61^2 = 0.372$$

$$\pi_{UD} = \pi_{DU} = 0.61 \times 0.39 = 0.238$$

$$\pi_{DD} = 0.39^2 = 0.152$$

c_{UU} = \$44
c_{UD} = \$18
c_{DU} = \$18
c_{DD} = \$0

$$c_t = \frac{(0.372 \times \$44) + (0.238 \times \$18) + (0.238 \times \$18) + (0.152 \times \$0)}{e^{(0.039) \times 2}} = \$23.07$$

Professor's Note: You may have a slightly different result due to rounding. Focus on the mechanics of the calculation.

2. A put = call – stock + (exercise price × e^{-rT})

   = \$23.07 – \$80 + [\$62 × $e^{-(0.039)(2)}$] = \$0.42

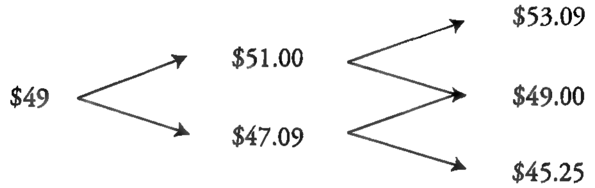
3. C The up-move factor is $U = e^{\sigma\sqrt{t}} = e^{0.08\sqrt{0.25}} = 1.041$.

   The down-move factor is $D = \frac{1}{U} = \frac{1}{1.041} = 0.961$.

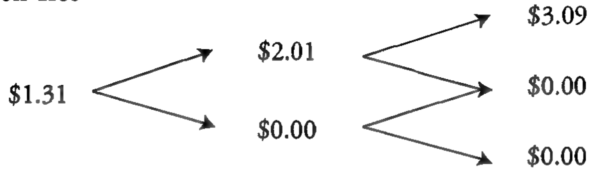
   The probability of an up move in JTE stock $= \frac{e^{rt} - D}{U - D} = \frac{e^{0.055(0.25)} - 0.961}{1.041 - 0.961} = 0.66$.

   The probability of a down move in JTE = 1 – 0.66 = 0.34.

Stock Tree



Option Tree



The $50 call option is in-the-money when the stock price finishes at $53.09 and the call has a value of $3.09.

The present value of the expected payoff in the up node at the end of three months is:

$$\frac{(\$3.09\times0.66)+(\$0\times0.34)}{e^{0.055\times0.25}}=\$2.01$$

Since this is an American option, we need to see if it is optimal to exercise the option early. The payoff from early exercise in the up node of the first 3-month period is max ($51 – 50, 0) = $1.00. Since $1.00 < $2.01, it is not optimal to exercise the option early.

The value of the option today is calculated as:

$$\frac{(\$2.01\times0.66)+(\$0\times0.34)}{e^{0.055\times0.25}}=\$1.31$$

4. A The payoff from exercising the option is the exercise price minus the current stock price: $50 – $45 = $5. The discounted value of the expected future payoff is:

$$\frac{(0\times0.55)+(8\times0.45)}{e^{(0.03)\times1}}=\$3.49$$

It is optimal to exercise the option early because it is worth more exercised ($5.00) than if not exercised ($3.49).

5. A First calculate the size of the up- and down-move factors:

$$U=e^{\sigma\sqrt{t}}=e^{0.03\sqrt{1}}=1.03$$

$$D=\frac{1}{U}=\frac{1}{1.03}=0.97$$

The risk-neutral probability of an up move is calculated as follows:

$$\pi_u=\frac{e^{(r-q)t}-D}{U-D}=\frac{e^{(0.03-0.02)}-0.97}{1.03-0.97}=\frac{0.04}{0.06}=0.67$$

The following is a review of the Valuation and Risk Models principles designed to address the AIM statements set forth by GARP®. This topic is also covered in:

The Black-Scholes-Merton Model

Topic 41

Exam Focus

The Black-Scholes-Merton (BSM) option pricing model (often referred to as the Black-Scholes model) is based on the assumption that stock prices are lognormally distributed. In this topic, we examine the calculation of call and put options using the BSM option pricing model. Also, we discuss how volatility can be estimated using a combination of the BSM model and current option prices. For the exam, know how to calculate the value of a call and put option using the BSM model and be able to incorporate dividends into the model if necessary. Put-call parity can be applied to calculate call or put values since the BSM model requires the use of European options.

AIM 41.1: Explain the lognormal property of stock prices, the distribution of rates of return, and the calculation of expected return.

AIM 41.2: Compute the realized return and historical volatility of a stock.

Lognormal Stock Prices

The model used to develop the **Black-Scholes-Merton** (BSM) **model** assumes stock prices are lognormally distributed.

Equation 1

$$\ln S_T \sim N\left[\ln S_0 + \left(\mu - \frac{\sigma^2}{2}\right)T, \sigma\sqrt{T}\right]$$

where:
S_T = stock price at time T
S_0 = stock price at time 0
μ = expected return on stock per year
σ = volatility of the stock price per year
$N[m, s]$ = normal distribution with mean = m and standard deviation = s

Since ln S_T is normally distributed, S_T has a lognormal distribution.

Example: Calculating mean and standard deviation

Assume a stock has an initial price S_0 = \$25, an expected annual return of 12%, and an annual volatility of 20%. **Calculate** the mean and standard deviation of the distribution of the stock price in three months.

Answer:

Using Equation 1, the probability distribution of the stock price, S_T, in three months would be:

$$\ln S_T \sim N\left\{\ln 25 + \left[\left(0.12 - \frac{0.2^2}{2}\right) \times 0.25\right], 0.2 \times \sqrt{0.25}\right\}$$

$$\ln S_T \sim N(3.244,\ 0.10)$$

Since ln S_T is normally distributed, 95% of the values will fall within 1.96 standard deviations of the mean. Therefore, ln S_T will lie between 3.244±(1.96 × 0.1). Stated another way:

$$e^{3.244-1.96\times0.1} < S_T < e^{3.244+1.96\times0.1}$$

$$21.073 < S_T < 31.187$$

Dividing the mean and standard deviation in Equation 1 by T results in the continuously compounded annual return of a stock price. Specifically, the continuously compounded annual returns are *normally distributed* with a mean of:

$$\left(\mu - \frac{\sigma^2}{2}\right)$$

and a standard deviation of:

$$\frac{\sigma}{\sqrt{T}}$$

Professor's Note: Notice that the BSM model assumes stock prices are lognormally distributed, but stock returns are normally distributed. Also, notice in the standard deviation formula that volatility will be lower for longer periods of time.

Example: Return distribution

Assume a stock has an expected annual return of 12% and an annual volatility of 20%. **Calculate** the mean and standard deviation of the probability distribution for the continuously compounded average rate of return over a 4-year period.

Answer:

$$\text{mean} = 0.12 - \frac{0.2^2}{2} = 0.10$$

$$\text{standard deviation} = \frac{0.2}{\sqrt{4}} = 0.10$$

Expected Value

Using the properties of a *lognormal distribution*, we can show that the expected value of S_T, $E(S_T)$, is:

$$E(S_T) = S_0 e^{\mu T}$$

where:
μ = expected rate of return

Example: Expected stock price

Assume a stock is currently priced at $25 with an expected annual return of 20%. **Calculate** the expected value of the stock in six months.

Answer:

$$E(S_T) = \$25 \times e^{0.2 \times 0.5} = \$27.63$$

The difference between the expected annual return on a stock, μ, and the mean return, $\left[\mu - \frac{\sigma^2}{2}\right]$, is closely related to the difference between the arithmetic return and the geometric return. The mean return will always be slightly less than the expected return, just as the geometric return will always be slightly less than the arithmetic return.

When computing the **realized return** for a portfolio, we want to chain-link the returns just like in the calculation of a geometric mean. Using a geometric return produces a more accurate representation of portfolio return.

Example: Realized return

Consider a portfolio that has the following asset returns: 5%, –4%, 9%, 6%. **Calculate** the return realized by this portfolio.

Answer:

$$\text{realized portfolio return} = (1.05 \times 0.96 \times 1.09 \times 1.06)^{1/4} - 1 = 3.9\%$$

Estimating Historical Volatility

As we saw in the value at risk (VaR) topics in Book 2, the volatility for short periods of time can be scaled to longer periods in time. For example, if the weekly standard deviation is 5%, and we want the annual standard deviation, we simply scale it by the square root of the number of periods in a year, or $\sqrt{52}$. So the annual standard deviation in this case is 36.06%. Conversely, if we knew that the annual standard deviation was 36.06%, then the weekly standard deviation can be found using this formula: 36.06% / $\sqrt{52}$, which is 5%.

The volatility estimation process is a matter of collecting daily price data and then computing the standard deviation of the series of corresponding continuously compounded returns. Continuously compounded returns can be calculated as: $\ln(S_i / S_{i-1})$. The annualized volatility is simply the estimated volatility multiplied by the square root of the number of trading days in a year. Typically, 90 to 180 trading days of data is sufficient for this estimation technique, but a common rule of thumb is to use data covering a period equal to the length of the projection period. In other words, to estimate the volatility for the next year, we should use a year's worth of historical data.

Professor's Note: We will examine the calculation for historical volatility later in this topic.

Black-Scholes-Merton Model Assumptions

AIM 41.3: List and describe the assumptions underlying the Black-Scholes-Merton option pricing model.

The Black-Scholes-Merton model values options in continuous time and is derived from the same no-arbitrage assumption used to value options with the binomial model. In the binomial model, the hedge portfolio is riskless over the next period, and the no-arbitrage option price is the one that ensures that the hedge portfolio will yield the risk-free rate. To derive the BSM model, an "instantaneously" riskless portfolio (one that is riskless over the next instant) is used to solve for the option price based on the same logic.

In addition to the no-arbitrage condition, the assumptions underlying the BSM model are the following:

- **The price of the underlying asset follows a lognormal distribution.** A variable that is lognormally distributed is one where the logs of the values (in this case, the continuous returns) are normally distributed. It has a minimum of zero and conforms to prices better than the normal distribution (which would produce negative prices).
- **The (continuous) risk-free rate is constant and known.**
- **The volatility of the underlying asset is constant and known.** Option values depend on the volatility of the price of the underlying asset or interest rate.
- **Markets are "frictionless."** There are no taxes, no transactions costs, and no restrictions on short sales or the use of short-sale proceeds.
- **The underlying asset has no cash flow,** such as dividends or coupon payments.
- **The options valued are European options,** which can only be exercised at maturity. The model does not correctly price American options.

Black-Scholes-Merton Option Pricing Model

AIM 41.4: Compute the value of a European option using the Black-Scholes-Merton model on a non-dividend-paying stock.

The formulas for the BSM model are:

$$c_0 = \left[S_0 \times N(d_1)\right] - \left[X \times e^{-R_f^c \times T} \times N(d_2)\right]$$

$$p_0 = \left\{X \times e^{-R_f^c \times T} \times \left[1 - N(d_2)\right]\right\} - \left\{S_0 \times \left[1 - N(d_1)\right]\right\}$$

where:

$$d_1 = \frac{\ln\left(\frac{S_0}{X}\right) + \left[R_f^c + \left(0.5 \times \sigma^2\right)\right] \times T}{\sigma \times \sqrt{T}}$$

$$d_2 = d_1 - \left(\sigma \times \sqrt{T}\right)$$

T = time to maturity (as % of a 365-day year)
S_0 = asset price
X = exercise price
R_f^c = continuously compounded risk-free rate
σ = volatility of continuously compounded returns on the stock
$N(\bullet)$ = cumulative normal probability

We've given you the formulas for both call and put values. However, remember that if you're given one of those prices, you can always use **put-call parity** (with continuously compounded interest rates) to calculate the other one:

$$c_0 = p_0 + S_0 - \left(X \times e^{-R_f^c \times T}\right)$$

or

$$p_0 = c_0 - S_0 + \left(X \times e^{-R_f^c \times T}\right)$$

$N(d_1)$ and $N(d_2)$ are found in a table of probability values (i.e., the *z*-table), so any question about the value of an option will provide those values. The rest is a straightforward calculation.

Example: Using the Black-Scholes-Merton model to value a European call option

Suppose that the stock of Vola, Inc., is trading at $50, and there is a call option on Vola available with an exercise price of $45 that expires in three months. The continuously compounded risk-free rate is 5%, and the annualized standard deviation of returns is 12%. Using the Black-Scholes-Merton model, **calculate** the value of the call option.

Answer:

First we must compute d_1 and d_2 as follows:

$$d_1 = \frac{\ln\left(\frac{50}{45}\right) + \left[0.05 + \left(0.5 \times 0.12^2\right)\right] \times 0.25}{0.12 \times \sqrt{0.25}} = 1.99$$

$$d_2 = 1.99 - \left(0.12 \times \sqrt{0.25}\right) = 1.93$$

Now look up these values in the normal probability tables in Figure 1.

Figure 1: Partial Cumulative Normal Distribution Table*

	0.03	0.04	0.05	0.06	0.07	0.08	0.09
1.8	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817

* Note: This table is incomplete. To view an example of a complete cumulative normal table, see the table included at the back of this book.

From the table, we determine that $N(d_1)$ is 0.9767 and $N(d_2)$ is 0.9732. Now that we have everything we need to apply the main call option formula, the value of the call is:

$$c_0 = (\$50 \times 0.9767) - \left(\$45 \times e^{-(0.05 \times 0.25)} \times 0.9732\right) = \$48.84 - \$43.25 = \$5.59$$

To price the corresponding put option using the data in our example, we simply solve put-call parity for the put option price.

Example: Calculating put option value

Calculate the value of a Vola 3-month put option with an exercise price of \$45, given the information in the previous example.

Answer:

We can use put-call parity to find the value of the comparable put:

$$p_0 = \$5.59 - \$50.00 + [\$45.00 \times e^{-(0.05 \times 0.25)}] = \$0.03$$

We can also use the BSM put formula:

$$p_0 = \left[\$45 \times e^{-0.05 \times 0.25} \times (1 - 0.9732)\right] - \left[\$50 \times (1 - 0.9767)\right] = \$0.03$$

Professor's Note: You should know how to look up $N(d_1)$ and $N(d_2)$ in the normal probability table given d_1 and d_2. It's possible, however unlikely, that you will have to calculate d_1 and d_2 without the formulas. To value the put option, memorize put-call parity and use it to solve for the put value given the call value.

Black-Scholes-Merton Model With Dividends

AIM 41.8: Compute the value of a European option using the Black-Scholes-Merton model on a dividend-paying stock.

European Options

Just as we subtracted the present value of expected cash flows from the asset price when valuing forwards and futures, we can subtract it from the asset price in the BSM model. Since the BSM model is in continuous time, in practice $S_0 \times e^{-qT}$ is substituted for S_0 in the BSM formula, where q is equal to the continuously compounded rate of dividend payment. Over time, the asset price is discounted by a greater amount to account for the greater amount of cash flows. Cash flows will increase put values and decrease call values.

Example: Valuing a call option on a stock with a continuous dividend yield

Let's revisit Vola, Inc., and this time we'll assume the stock pays a continuous dividend yield of 2%. Here's the basic information again. Suppose the stock of Vola is trading at \$50, and there is a call option available with an exercise price of \$45 that expires in three months. The continuously compounded risk-free rate is 5%, and the annualized standard deviation of returns is 12%. Using the Black-Scholes-Merton model, **calculate** the value of the call option.

Answer:

The adjusted price of the stock is:

$$e^{-0.02 \times 0.25} \times \$50.00 = \$49.75$$

Recalculate d_1 and d_2 using the adjusted price:

$$d_1 = \frac{\ln\left(\frac{49.75}{45}\right) + \left\{0.05 + \left[0.5 \times (0.12)^2\right]\right\} \times 0.25}{0.12 \times \sqrt{0.25}} = 1.91$$

$$d_2 = 1.91 - \left(0.12 \times \sqrt{0.25}\right) = 1.85$$

Now look up these values in the normal probability tables in Figure 2.

Figure 2: Partial Cumulative Normal Distribution Table*

	0.00	0.01	0.02	0.03	0.04	0.05	0.06
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803

* Note: This table is incomplete. To view an example of a complete cumulative normal table, see the table included at the back of this book.

From the table, we can determine that $N(d_1)$ is 0.9719 and $N(d_2)$ is 0.9678. The adjusted price is \$49.75. Now that we have everything we need to apply the BSM model, the value of the call is:

$$c_0 = (\$49.75 \times 0.9719) - \left(\$45 \times e^{-(0.05 \times 0.25)} \times 0.9678\right) = \$48.35 - \$43.01 = \$5.34$$

The value of the Vola call with no dividend yield was \$5.59 from our earlier example. The 2% dividend yield reduced the call value by \$0.25, from \$5.59 to \$5.34.

On the exam, it may be the case that you are provided with the dollar amount of the dividend rather than the dividend yield. The process for computing option value is similar, but instead of discounting the stock price with a continuously compounded dividend rate, you would compute the present value of the dividend(s) and then subtract that amount from the stock price. The following example demonstrates this technique.

Example: Pricing options on a dividend paying stock

Assume we have a non-dividend paying stock with a current price of $100 and volatility of 20%. If the risk-free rate is 7%, the price of a 6-month at-the-money call option, according to the Black-Scholes-Merton model, will be $7.43, and the corresponding put option price will be $3.99. Now assume that the same stock instead pays a $1 dividend in two months and a $1 dividend in five months. **Compute** the value of a 6-month call option on the dividend paying stock.

Answer:

The present value of the first dividend is $1e^{-0.07(0.1667)} = 0.9884$, and the present value of the second dividend is $1e^{-0.07(0.4167)} = 0.9713$. The stock price then becomes:

$$S_0 = 100 - 0.9884 - 0.9713 = \$98.04$$

We now know the following: $S_0 = \$98.04$; $X = \$100$; $\sigma = 20\%$; $r = 7\%$; $T = 0.5$

d_1 and d_2 are computed as follows:

$$d_1 = \frac{\ln\left(\frac{98.04}{100}\right) + \left(0.07 + \frac{0.2^2}{2}\right) \times 0.5}{0.2\sqrt{0.5}} = 0.1783$$

$$d_2 = 0.1783 - 0.2\sqrt{0.5} = 0.03688$$

From the cumulative standard normal tables, we find:

$$N(d_1) = 0.5708 \text{ and } N(d_2) = 0.5147$$

Substituting back into the call option price formula yields:

$$c = 98.04 \times 0.5708 - 100e^{-0.07 \times 0.5} \times 0.5147 = \$6.26$$

Using put-call parity, the corresponding put option price is:

$$p = \$6.26 + 100e^{-0.07 \times 0.5} - 98.04 = \$4.78$$

Since the dividend reduces the value of the stock, the call value decreased, and the put value increased compared to the non-dividend paying stock.

AIM 41.7: Explain how dividends affect the early exercise decision for American call and put options.

AIM 41.9: Use Black's Approximation to compute the value of an American call option on a dividend-paying stock.

American Options

Recall that when no dividends are paid, there is no difference between European and American call options. This is because the unexercised value of a call option, $S_0 - Xe^{-rT}$, was always more valuable than the exercised value of the option, $S_0 - X$. When a stock pays a dividend, *D*, at time *n*, the exercise decision becomes more complicated.

At the last dividend date before expiration, t_n, the exercised value of the option is:

$$S(t_n) - X$$

If the call option is unexercised and the dividend is paid, its unexercised value is:

$$S(t_n) - D_n - Xe^{-r(T-t_n)}$$

An investor will only exercise when:

$$S(t_n) - X > S(t_n) - D_n - Xe^{-r(T-t_n)}$$

or

$$D_n > X\left(1 - e^{-r(T-t_n)}\right)$$

So the closer the option is to expiration and the larger the dividend, the more optimal early exercise will become. The previous result can be generalized to show that early exercise is not optimal if:

$$D_i \le X\left(1 - e^{-r(t_{i+1}-t_n)}\right) \text{for } i < n$$

A popular approximation for pricing American call options on dividend paying stocks is **Black's approximation**. Black suggests using the procedure for European options on *T* and t_n and then taking the larger of the two as the price of the American call option.

Consider the situation provided in the previous example. However, instead of evaluating a European option, assume the call option is an American option. We know that the call option value at maturity, T = 6 months, with dividend payments at two months and five months was $6.26. Suppose an investor instead opted to exercise the option immediately

before the second dividend payment. Here, exercise may be optimal, if the option is deep in-the-money, because the second $1 dividend, D_2, is greater than 0.5816.

$$1 > \$100\left(1-e^{-0.07\times\left(\frac{6}{12}-\frac{5}{12}\right)}\right) = 0.5816$$

In this case, we can apply Black's approximation by computing the call option value assuming early exercise before the second dividend payment. When only considering the first dividend's present value of 0.9884, the stock price becomes $99.0116. The call option value, according to the Black-Scholes-Merton model, is now $6.05. Since Black's approximation values the American option as the greater of the two values ($6.26 > $6.05), we would value this option at $6.26.

For American put options, early exercise becomes less likely with larger dividends. The value of the put option increases as the dividend is paid. Early exercise is, therefore, not optimal as long as:

$$D_n \geq X\left(1-e^{-r(T-t_n)}\right)$$

Valuation of Warrants

AIM 41.5: Identify the complications involving the valuation of warrants.

Warrants are attachments to a bond issue that give the holder the right to purchase shares of a security at a stated price. After purchasing the bond, warrants can be exercised separately or stripped from the bond and sold to other investors. Hence, warrants can be valued as a separate call option on the firm's shares. One distinction is necessary though. With call options, the shares are already outstanding, and the exercise of a call option triggers the trading of shares among investors at the strike price of the call options. When an investor exercises warrants, the investor purchases shares directly from the firm. The distinction is that the value of all outstanding shares can be affected by the exercise of warrants, as the amount paid for the shares will (in all likelihood) be less than their pro-rata market value, so the value of equity per share will fall with exercise (i.e., dilution can occur).

After issue, bonds may trade with or without warrants attached. When both are actively traded, the value of the warrants can be easily determined by the difference between the market prices of the two instruments.

Volatility Estimation

AIM 41.6: Define implied volatilities and describe how to compute implied volatilities from market prices of options using the Black-Scholes-Merton model.

Notice in call and put equations that volatility is unobservable. Historical data can serve as a basis for what volatility might be going forward, but it is not always representative of the current market. Consequently, practitioners will use the BSM option pricing model along with market prices for options and solve for volatility. The result is what is known

as **implied volatility**. Before we discuss implied volatility further, let's first examine the calculation of historical volatility.

The steps in computing **historical volatility** for use as an input in the BSM continuous-time options pricing model are:

- Convert a time series of *N* prices to returns:

$$R_i = \frac{P_i - P_{i-1}}{P_{i-1}}, \, i = 1 \text{ to } N$$

- Convert the returns to continuously compounded returns:

$$R_i^c = \ln(1 + R_i), \, i = 1 \text{ to } N$$

- Calculate the variance and standard deviation of the continuously compounded returns:

$$\sigma^2 = \frac{\sum_{i=1}^{N}\left(R_i^c - \overline{R}_i^c\right)^2}{N-1}$$

$$\sigma = \sqrt{\sigma^2}$$

Recall from Book 2 that continuously compounded returns can be calculated using a set of price data. We introduced the equation for continuously compounded returns as:

$$u_i = \ln\left(\frac{S_i}{S_{i-1}}\right)$$

Arriving at the continuously compounded return value is no different than taking the holding period return and then taking the natural log of (1 + holding period return) as illustrated above. For example, if we assume that a stock price is currently valued at \$50 and was \$47 yesterday, the continuously compounded return can be computed as either:

$$u_i = \ln\left(\frac{S_i}{S_{i-1}}\right) = \ln\left(\frac{50}{47}\right) = 6.19\%$$

or

$$R_i = \frac{P_i - P_{i-1}}{P_{i-1}} = \frac{50 - 47}{47} = 6.38\%$$

$$R_i^c = \ln(1 + 0.0638) = 6.19\%$$

Implied volatility is the value for standard deviation of continuously compounded rates of return that is "implied" by the market price of the option. Of the five inputs into the BSM model, four are observable: (1) stock price, (2) exercise price, (3) risk-free rate, and (4) time to maturity. If we use these four inputs in the formula and set the BSM formula equal to market price, we can solve for the volatility that satisfies the equality.

Volatility enters into the equation in a complex way, and there is no closed-form solution for the volatility that will satisfy the equation. Rather, it must be found by iteration (trial and error). If a value for volatility makes the value of a call calculated from the BSM model lower than the market price, it needs to be increased (and vice versa) until the model value equals market price (remember, option value and volatility are positively related).

Volatility Index

The most widely used index for volatility is the Chicago Board Options Exchange Volatility Index (ticker symbol: VIX). The VIX demonstrates implied volatility on wide variety of calls and puts on the S&P 500 index. Note that trading in futures and options on the VIX is a bet on volatility only. Since inception, the VIX has displayed an average of around 20 (which corresponds to volatility of 20% on the S&P 500 index options), but reached an intraday high in October 2008 of close to 90.

Key Concepts

1. The Black-Scholes-Merton model suggests that stock prices are lognormal over longer periods of time, but suggests that stock returns are normally distributed.

2. Assumptions underlying the BSM model:
   - The price of the underlying asset follows a lognormal distribution.
   - The (continuous) risk-free rate is constant and known.
   - The volatility of the underlying asset is constant and known.
   - Markets are "frictionless."
   - The underlying asset generates no cash flows.
   - The options are European.

3. The formulas for the BSM model are:

$$c_0 = \left[S_0 \times N(d_1)\right] - \left[X \times e^{-R_f^c \times T} \times N(d_2)\right]$$

$$p_0 = \left\{X \times e^{-R_f^c \times T} \times \left[1 - N(d_2)\right]\right\} - \left\{S_0 \times \left[1 - N(d_1)\right]\right\}$$

4. Cash flows on the underlying asset decrease call prices and increase put prices. To adjust the BSM model for assets with a continuously compounded rate of dividend payment equal to q, $S_0 \times e^{-q \times T}$ is substituted for S_0 in the formula.

5. Dividends complicate the early exercise decision for American-style options because a dividend payment effectively decreases the price of the stock.

6. Historical volatility is the standard deviation of a past series of continuously compounded returns for the underlying asset. Implied volatility is the volatility that, when used in the Black-Scholes-Merton formula, produces the current market price of the option.

Concept Checkers

1. A European put option has the following characteristics: S_0 = \$50; X = \$45; r = 5%; T = 1 year; and σ = 25%. Which of the following is closest to the value of the put?
   A. \$1.88.
   B. \$3.28.
   C. \$9.07.
   D. \$10.39.

2. A European call option has the following characteristics: S_0 = \$50; X = \$45; r = 5%; T = 1 year; and σ = 25%. Which of the following is closest to the value of the call?
   A. \$1.88.
   B. \$3.28.
   C. \$9.06.
   D. \$10.39.

3. A security sells for \$40. A 3-month call with a strike of \$42 has a premium of \$2.49. The risk-free rate is 3%. What is the value of the put according to put-call parity?
   A. \$1.89.
   B. \$3.45.
   C. \$4.18.
   D. \$6.03.

4. Which of the following is not an assumption underlying the BSM options pricing model?
   A. The underlying asset does not generate cash flows.
   B. Continuously compounded returns are lognormally distributed.
   C. The option can only be exercised at maturity.
   D. The risk-free rate is constant.

5. Stock ABC trades for \$60 and has 1-year call and put options written on it with an exercise price of \$60. The annual standard deviation estimate is 10%, and the continuously compounded risk-free rate is 5%. The value of both the call and put using the BSM option pricing model are closest to:

	Call	Put
A.	\$6.21	\$1.16
B.	\$4.09	\$3.28
C.	\$4.09	\$1.16
D.	\$6.21	\$3.28

Concept Checker Answers

1. **A** S_0 = \$50; X = \$45; r = 5%; T = 1 year; and σ = 25%.

$$d_1 = \frac{\ln\left(\frac{50}{45}\right) + \left(0.05 + \frac{0.0625}{2}\right)1}{0.25(1)} = \frac{0.18661}{0.25} = 0.74644$$

$d_2 = 0.74644 - 0.25 = 0.49644$

from the cumulative normal table:

$N(-d_1) = 0.2266$

$N(-d_2) = 0.3085$*

$p = 45e^{-0.05(1)}(0.3085) - 50(0.2266) = 1.88$

(*note rounding)

2. **C** S_0 = \$50; X = \$45; r = 5%; T = 1 year; and σ = 25%.

$$d_1 = \frac{\ln\left(\frac{50}{45}\right) + \left(0.05 + \frac{0.0625}{2}\right)1}{0.25(1)} = \frac{0.18661}{0.25} = 0.74644$$

$d_2 = 0.74644 - 0.25 = 0.49644$

from the cumulative normal table:

$N(d_1) = 0.7731$

$N(d_2) = 0.6915$*

$c = 50(0.7731) - 45e^{-0.05}(0.6915) = 9.055$

(*note rounding)

3. **C** $p = c + Xe^{-rT} - S = 2.49 + 42\,e^{-0.03 \times 0.25} - 40 = \$4.18$

4. **B** No arbitrage is possible, and:
   - Asset price (not returns) follows a lognormal distribution.
   - The (continuous) risk-free rate is constant.
   - The volatility of the underlying asset is constant.
   - Markets are "frictionless."
   - The asset has no cash flows.
   - The options are European (i.e., they can only be exercised at maturity).

5. C First, let's compute d_1 and d_2 as follows:

$$d_1 = \frac{\ln\left(\frac{60}{60}\right) + \left[0.05 + \left(0.5 \times 0.10^2\right)\right] \times 1.0}{0.1 \times \sqrt{1.0}} = 0.55$$

$$d_2 = 0.55 - \left(0.1 \times \sqrt{1.0}\right) = 0.45$$

Now look up these values in the normal table at the back of this book. These values are $N(d_1) = 0.7088$ and $N(d_2) = 0.6736$. Hence, the value of the call is:

$$c_0 = \$60(0.7088) - \left[\$60 \times e^{-(0.05 \times 1.0)} \times (0.6736)\right] = \$42.53 - \$38.44 = \$4.09$$

According to put/call parity, the put's value is:

$$p_0 = c_0 - S_0 + \left(X \times e^{-R_c^f \times T}\right) = \$4.09 - \$60.00 + \left[\$60.00 \times e^{-(0.05 \times 1.0)}\right] = \$1.16$$

The following is a review of the Valuation and Risk Models principles designed to address the AIM statements set forth by GARP®. This topic is also covered in:

The Greek Letters

Topic 42

Exam Focus

The level of risk associated with an option position is dependent in large part upon the following factors: relationship between the value of a position involving options and the value of the underlying assets; time until expiration; asset value volatility; and the risk-free rate. Measures that capture the effects of these factors are referred to as "the Greeks" due to their names: delta; theta; gamma; vega; and rho. Thus, a large part of this topic covers the evaluation of option Greeks. Once option participants are aware of their Greek exposures, they can more effectively hedge their positions to mitigate risk. This topic also introduces the common hedging concepts of delta-neutral portfolios and portfolio insurance.

Naked and Covered Call Options

AIM 42.1: Discuss and assess the risks associated with naked and covered option positions.

A **naked position** occurs when one party sells a call option without owning the underlying asset. A **covered position** occurs when the party selling a call option owns the underlying asset.

Suppose a firm can sell 10,000 call options on a stock that is currently trading at \$20. The strike price of the option is \$23, and the option premium is \$4. A naked position would generate \$40,000 in revenue, and as long as the stock price is below \$23 at expiration, the firm can retain the income without cost. However, the initial income will be reduced by \$10,000 for every dollar above \$23 that the stock reaches at expiration. For example, if the stock is at \$30 per share when the option expires, the naked position results in a negative payoff of \$70,000 and a net loss of \$30,000. The potential loss from a naked written position is unlimited, assuming the stock's price can rise without bound. The maximum potential gain is capped at the level of the premium received. If the stock price at expiration is \$23 or less, the writer makes a profit equal to the premium of \$40,000.

With a covered call, the firm owns 10,000 shares of the underlying stock, so if the stock price rises above the \$23 strike price and the option is exercised, the firm will sell shares that it already owns. This minimizes the "cost" of the short options by locking in the revenue from the option sale. However, if the stock falls to \$10 per share, the long stock position decreases in value by \$100,000, which is substantially larger than the premium received from the option sale.

A Stop-Loss Strategy

AIM 42.2: Explain how naked and covered option positions generate a stop-loss trading strategy.

Stop-loss strategies with call options are designed to limit the losses associated with short option positions (i.e., those taken by call writers). The strategy requires purchasing the underlying asset for a naked call position when the asset rises above the option's strike price. The asset is then sold as soon as it goes below the strike price. The objective here is to hold a naked position when the option is out-of-the-money and a covered position when the option is in-the-money.

The main drawbacks to this simplistic approach are transaction costs and price uncertainty. The costs of buying and selling the asset can become high as the frequency of stock price fluctuations about the exercise price increases. In addition, there is great uncertainty as to whether the asset will be above (or below) the strike price at expiration.

Delta Hedging

AIM 42.3: Define delta hedging for an option, forward, and futures contracts.

AIM 42.4: Compute delta for an option.

The **delta** of an option, Δ, is the ratio of the change in price of the call option, c, to the change in price of the underlying asset, s, for small changes in s. Mathematically:

$$\text{delta} = \Delta = \frac{\partial c}{\partial s}$$

where:
∂c = change in the call option price
∂s = change in the stock price

As illustrated in Figure 1, delta is the slope of the call option pricing function at the current stock price. As shown in Figure 2, call option deltas range from zero to positive one, while put option deltas range from negative one to zero.

Figure 1: Delta of a Call Option

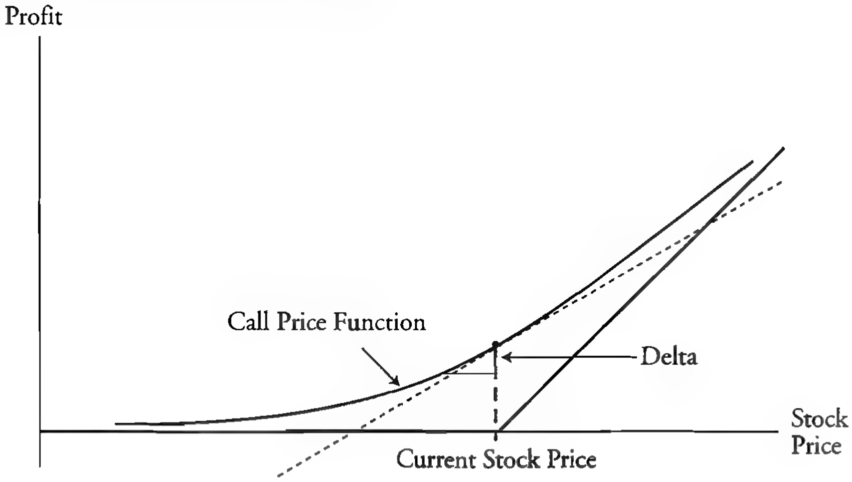
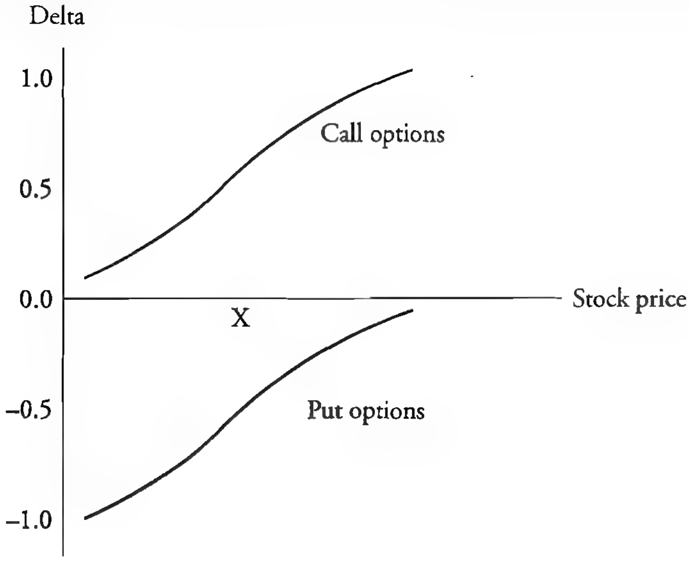


Figure 2: Call and Put Option Deltas



Option Delta

A call delta equal to 0.6 means that the price of a call option on a stock will change by approximately \$0.60 for a \$1.00 change in the value of the stock. To completely hedge a long stock or short call position, an investor must purchase the number of shares of stock equal to delta times the number of options sold. Another term for being completely hedged is **delta neutral**. For example, if an investor is short 1,000 call options, he will need to be long 600 (0.6 × 1,000) shares of the underlying. When the value of the underlying asset increases by \$1.00, the underlying position increases by \$600, while the value of his option position decreases by \$600. When the value of the underlying asset decreases by \$1.00, there is an offsetting increase in value in the option position.

Delta can also be calculated as the $N(d_1)$ in the Black-Scholes-Merton option pricing model. Recall from the previous topic that d_1 is equal to:

$$d_1 = \frac{\ln(S_0 / X) + (R_F + \sigma^2 / 2) \times T}{\sigma \times \sqrt{T}}$$

Example: Computing delta

Suppose that stock XYZ is trading at $50, and there is a call option that trades on XYZ with an exercise price of $45 which expires in three months. The risk-free rate is 5% and the standard deviation of returns is 12% annualized. **Determine** the value of the call option's delta.

Answer:

$$d_1 = \frac{\ln(50 / 45) + (0.05 + 0.12^2 / 2) \times 0.25}{0.12 \times \sqrt{0.25}} = 1.99$$

Next, look up this value in the normal probability tables, which can be found in appendix at the end of this book. From the normal probability tables N(1.99), and, in turn, delta is 0.9767. This means that when the stock price changes by $1, the option price will change by 0.9767.

Forward Delta

The delta of a forward position is equal to one, implying a one-to-one relationship between the value of the forward contract and its underlying asset. A forward contract position can easily be hedged with an offsetting underlying asset position with the same number of securities.

Professor's Note: When the underlying asset pays a dividend, q, the delta of an option or forward must be adjusted. If a dividend yield exists, the delta for a call option equals $e^{-qT} \times N(d_1)$, the delta of a put option equals $e^{-qT} \times [N(d_1) - 1]$, and the delta of a forward contract equals e^{-qT}.

Futures Delta

Unlike forward contracts, the delta of a futures position is not ordinarily one because of the spot-futures parity relationship. For example, the delta of a futures position is e^{rT} on a stock or stock index that pays no dividends, where r is the risk-free rate and T is the time to maturity. Assets that pay a dividend yield, q, would generate a delta equal to $e^{(r-q)T}$. An investor would hedge short futures positions by going long the amount of the deliverable asset.

Dynamic Aspects of Delta Hedging

AIM 42.5: Discuss the dynamic aspects of delta hedging.

As we saw in Figure 1, the delta of an option is a function of the underlying stock price. That means when the stock price changes, so does the delta. When the delta changes, the portfolio will no longer be hedged (i.e., the number of options and underlying stocks will no longer be in balance), and the investor will need to either purchase or sell the underlying asset. This rebalancing must be done on a continual basis to maintain the delta-neutral hedged position.

The goal of a **delta-neutral portfolio** (or delta-neutral hedge) is to combine a position in an asset with a position in an option *so that the value of the portfolio does not change with changes in the value of the asset.* In referring to a stock position, a delta-neutral portfolio can be made up of a risk-free combination of a long stock position and a short call position where the number of calls to short is given by $1/\Delta_c$.

$$\text{number of options needed to delta hedge} = \frac{\text{number of shares hedged}}{\text{delta of call option}}$$

Example: Delta-neutral portfolio—Part 1

An investor owns 60,000 shares of ABC stock that is currently selling for \$50. A call option on ABC with a strike price of \$50 is selling at \$4 and has a delta of 0.60. **Determine** the number of call options necessary to create a delta-neutral hedge.

Answer:

In order to determine the number of call options necessary to hedge against instantaneous movements in ABC's stock price, calculate:

$$\text{number of options needed to delta hedge} = \frac{60{,}000}{0.6} = 100{,}000 \text{ options}$$

$$= 1{,}000 \text{ call option contracts}$$

Because he is long the stock, he needs to short the options.

Example: Delta-neutral portfolio—Part 2

Calculate the effect on portfolio value of a \$1.00 increase in the price of ABC stock.

Answer:

Assuming the price of ABC stock increased instantly by \$1.00, then the value of the call option position would decrease by \$0.60 because the investor is *short* (or has sold) the call option contracts. Therefore, the net impact of the price change would be zero as illustrated here:

total value of increase in stock position = (60,000) × (\$1) = \$60,000

total value of decrease in option position = (100,000) × (–\$0.60)= –\$60,000

total change in portfolio value = \$60,000 – \$60,000 = \$0

Recall that when short a call (or other asset), as the price of the underlying rises, the position loses value, and when the price of the underlying declines, the value of the position increases.

Maintaining the Hedge

A key consideration in delta-neutral hedging is that the *delta-neutral position only holds for very small changes in the value of the underlying stock.* Hence, the delta-neutral portfolio must be frequently (continuously) *rebalanced to maintain the hedge.* As the underlying stock price changes, so does the delta of the call option. The delta of the option is an approximation of a nonlinear function: the change in value of the option that corresponds with a change in the value of the underlying asset. As the delta changes, the number of calls that need to be sold to maintain a risk-free position also changes. Hence, continuously maintaining a delta-neutral position can be very costly in terms of transaction costs associated with either closing out options or selling additional contracts.

Example: Delta-neutral portfolio—Part 3

Continuing with the previous example, assume now that the price of the underlying stock has moved to $51, and consequently, the delta of the call option with a strike price of $50 has increased from 0.60 to 0.62. How would the investor's portfolio of stock and options have to be adjusted to maintain the delta-neutral position?

Answer:

In order to determine the number of call options necessary to maintain the hedge against instantaneous movements in ABC's stock price, recalculate the number of short call options needed:

$$\text{number of options needed} = \frac{\text{number of shares hedged}}{\text{delta of call option}} = \frac{60{,}000}{0.62} = 96{,}774$$

She will need 96,774 call options, or approximately 968 option contracts. In other words, 32 option contracts would need to be purchased in order to maintain the delta-neutral position. If the hedge were not modified, then another price change would result in a greater movement in the value of the options than in the underlying stock. With the rebalanced hedge, the change in value of her stock position will again be offset by the change in value of her short position. Assume the price of ABC stock increased (decreased) instantly by $1.00, then the value of the short call option position would decrease (increase) by $0.62. Therefore, the net impact of the price change would be zero:

increase in stock position = (60,000) × ($1) = $60,000

decrease in short position = (96,774) × (–$0.62)= –$60,000

Other Portfolio Hedging Approaches

It's also possible to develop a delta-neutral hedge by buying put options in sufficient numbers so that the current gain or loss on the underlying asset is offset by the current gain or loss on the puts. Hence, similar to the discussion of delta-neutral portfolios using call options, a delta-neutral position can be created by *purchasing* the correct number of put options so that:

Δ value of puts = – Δ value of long stock position

When using puts in constructing a delta-neutral portfolio, *purchase* [1 / (call delta – 1)] put options to protect a share of stock held long. When using calls you would *sell* (1 / call delta) call options for each long share of stock. Rebalancing is just as important with puts as it is with calls.

Example: Delta-neutral portfolio—Part 4

Using our earlier example, assume the investor owns 60,000 shares of ABC stock that is currently selling for \$50. A call option on ABC with a strike price of \$50 is selling at \$4 and has a delta of 0.60. **Determine** the number of put options necessary to create a delta-neutral hedge.

Answer:

First, compute the delta of the put option. The investor knows that the delta of a call option is 0.60. The delta of the put option is then equal to (call delta – 1), or (0.60 – 1) = –0.40. In order to determine the number of call options necessary to hedge against instantaneous movements in ABC's stock price, calculate:

$$\text{number of options needed to delta hedge} = \frac{-60{,}000}{-0.4} = 150{,}000 \text{ options}$$

$$= 1{,}500 \text{ put option contracts}$$

Because he is long the stock, he needs to purchase the put options.

AIM 42.6: Define the delta of a portfolio.

The delta of a portfolio of options on a single underlying asset can be calculated as the weighted average delta of each option position in the portfolio:

$$\text{portfolio delta} = \Delta_p = \sum_{i=1}^{n} w_i \Delta_i$$

where:
w_i = the portfolio weight of each option position
Δ_i = the delta of each option position

Therefore, portfolio delta represents the expected change of the overall option portfolio value given a small change in the price of the underlying asset.

THETA, GAMMA, VEGA, AND RHO

AIM 42.7: Define and describe theta, gamma, vega, and rho for option positions.

AIM 42.8: Explain how to implement and maintain a gamma-neutral position.

AIM 42.9: Discuss the relationship between delta, theta, and gamma.

THETA

Theta, Θ, measures the option's sensitivity to a decrease in time to expiration. Theta is also termed the "time decay" of an option. Theta varies as a function of both time and the price of the underlying asset. Figure 3 illustrates theta as a function of stock price and days until expiration.

Theta for a call option is calculated using the following equation:

$$\Theta = \frac{\partial c}{\partial t}$$

where:
∂c = change in the call price
∂t = change in time

For European call options on non-dividend-paying stocks, theta can be calculated using the Black-Scholes-Merton formula as follows:

$$\Theta(\text{call}) = -\frac{S_0 N'(d_1)\sigma}{2\sqrt{T}} - rXe^{-rT}N(d_2)$$

$$\Theta(\text{put}) = -\frac{S_0 N'(d_1)\sigma}{2\sqrt{T}} + rXe^{-rT}N(-d_2)$$

where:

$$N'(x) = \frac{1}{\sqrt{2\pi}}e^{-\left(x^2/2\right)}$$

Note that theta in the above equations is measured in years. It can be converted to a daily basis by dividing by 365. To find the theta for each trading day, you would divide by 252.

Example: Computing theta

Suppose that stock XYZ is trading at \$50, and there is a call option that trades on XYZ with an exercise price of \$45 which expires in three months. The risk-free rate is 5%, and the standard deviation of returns is 12% annualized. **Determine** the value of the call option's theta per trading day. Assume d_1 is 1.99 and d_2 is 1.93. From the normal probability tables, $N(d_1)$ is 0.9767 and $N(d_2)$ is 0.9732.

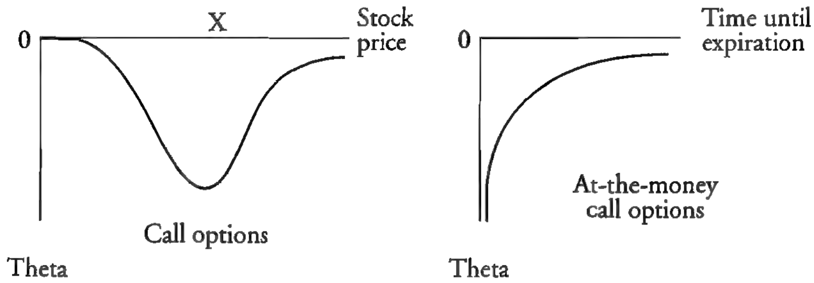
Answer:

$$N'(d_1) = \frac{1}{\sqrt{2\pi}} e^{-\left(1.99^2/2\right)} = 0.055$$

$$\Theta(\text{call}) = -\frac{50 \times 0.055 \times 0.12}{2\sqrt{0.25}} - 0.05 \times 45e^{-0.05 \times 0.25} \times 0.9732 = -0.33 - 2.16 = -2.49$$

Theta per trading day is: –2.49 / 252 = –0.00988

Figure 3: Theta as a Function of Stock Price and Time to Expiration



The specific characteristics of theta are as follows:

- Theta affects the value of put and call options in a similar way (e.g., as time passes, most call and put options decrease in value, all else equal).
- Theta varies with changes in stock prices and as time passes.
- Theta is most pronounced when the option is at-the-money, especially nearer to expiration. The left side of Figure 3 illustrates this relationship.
- Theta values are usually negative, which means the value of the option decreases as it gets closer to expiration.
- Theta usually increases in absolute value as expiration approaches. The right side of Figure 3 illustrates this relationship.
- It is possible for a European put option that is in-the-money to have a positive theta value.

GAMMA

Gamma, Γ, represents the expected change in the delta of an option. It measures the curvature of the option price function not captured by delta (see Figure 1). The specific mathematical relationship for gamma is:

$$\Gamma = \frac{\partial^2 c}{\partial s^2}$$

where:
$\partial^2 c$ and ∂s^2 = the second partial derivatives of the call and stock prices, respectively

The calculation of gamma for European call or put options on non-dividend-paying stocks can also be found using the following formula, where $N'(x)$ is calculated in the same fashion as it is for theta.

$$\Gamma = \frac{N'(d_1)}{S_0 \sigma \sqrt{T}}$$

Example: Computing gamma

Suppose that stock XYZ is trading at $50, and there is a call option that trades on XYZ with an exercise price of $45 which expires in three months. The risk-free rate is 5%, and the standard deviation of returns is 12% annualized. **Determine** the value of the call option's gamma. Assume d_1 is 1.99 and $N(d_1)$ is 0.9767.

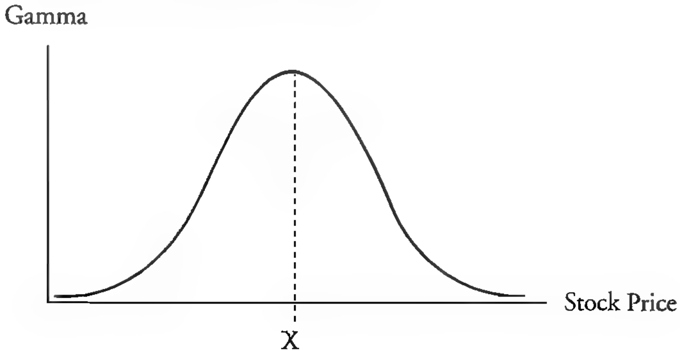
Answer:

$$\Gamma = \frac{N'(d_1)}{S_0 \sigma \sqrt{T}} = \frac{2.89}{50 \times 0.12 \times \sqrt{0.25}} = \frac{2.89}{3} = 0.96$$

Gamma measures the rate of change in the option's delta, so for a $1 change in the price of the stock, the delta will change by 0.96.

Figure 4 illustrates the relationship between gamma and the stock price for a stock option. As indicated in Figure 4, gamma is largest when an option is at-the-money (at stock price = X). When an option is deep in-the-money or out-of-the-money, changes in stock price have little effect on gamma.

Figure 4: Gamma vs. Stock Price



When gamma is large, delta will be changing rapidly. On the other hand, when gamma is small, delta will be changing slowly. Since gamma represents the curvature component of the call-price function not accounted for by delta, it can be used to minimize the *hedging error* associated with a linear relationship (delta) to represent the curvature of the call-price function.

Delta-neutral positions can hedge the portfolio against small changes in stock price, while gamma can help hedge against relatively large changes in stock price. Therefore, it is not only desirable to create a delta-neutral position but also to create one that is **gamma-neutral**. In that way, neither small nor large stock price changes adversely affect the portfolio's value.

Since underlying assets and forward instruments generate linear payoffs, they have zero gamma and, hence, cannot be employed to create gamma-neutral positions. Gamma-neutral positions have to be created using instruments that are not linearly related to the underlying instrument, such as options. The specific relationship that determines the number of options that must be added to an existing portfolio to generate a gamma-neutral position is $-(\Gamma_P / \Gamma_T)$, where Γ_P is the gamma of the existing portfolio position, and Γ_T is the gamma of a traded option that can be added. Let's take a look at an example.

Example: Creating a gamma-neutral position

Suppose an existing short option position is delta-neutral but has a gamma of –6,000. Here, gamma is negative because we are short the options. Also, assume that there exists a traded option with a delta of 0.6 and a gamma of 1.25. Create a gamma-neutral position.

Answer:

To gamma-hedge, we must buy 4,800 options (6,000 / 1.25). Now the position is gamma-neutral, but the added options have changed the delta position of the portfolio from 0 to 2,880 = 4,800 × 0.6. This means that 2,880 shares of the underlying position will have to be sold to maintain not only a gamma-neutral position, but also the original delta-neutral position.

Relationship Among Delta, Theta, and Gamma

Stock option prices are affected by delta, theta, and gamma as indicated in the following relationship:

$$r\Pi = \Theta + rS\Delta + 0.5\sigma^2S^2\Gamma$$

where:
r = the risk-neutral risk-free rate of interest
Π = the price of the option
Θ = the option theta
S = the price of the underlying stock
Δ = the option delta
σ^2 = the variance of the underlying stock
Γ = the option gamma

This equation shows that the change in the value of an option position is directly affected by its sensitivities to the Greeks.

For a delta-neutral portfolio, $\Delta = 0$, so the preceding equation reduces to:

$$r\Pi = \Theta + 0.5\sigma^2S^2\Gamma$$

The left side of the equation is the dollar risk-free return on the option (risk-free rate times option value). Assuming the risk-free rate is small, this demonstrates that for large positive values of theta, gamma tends to be large and negative, and vice versa, which explains the common practice of using theta as a proxy for gamma.

Vega

Professor's Note: Vega is not actually a letter of the Greek alphabet, but we still call vega one of the "Greeks" in option pricing.

Vega measures the sensitivity of the option's price to changes in the volatility of the underlying stock. For example, a vega of 8 indicates that for a 1% increase in volatility, the option's price will increase by 0.08. For a given maturity, exercise price, and risk-free rate, the vega of a call is equal to the vega of a put.

Vega for a call option is calculated using the following equation:

$$\text{vega} = \frac{\partial c}{\partial \sigma}$$

where:
∂c = change in the call price
$\partial \sigma$ = change in volatility

Vega for European calls and puts on non-dividend-paying stocks is calculated as:

$$\text{vega} = S_0 N'(d_1)\sqrt{T}$$

Example: Computing vega

Suppose that stock XYZ is trading at \$50, and there is a call option that trades on XYZ with an exercise price of \$45 which expires in three months. The risk-free rate is 5%, and the standard deviation of returns is 12% annualized. **Determine** the value of the call option's vega. Assume d_1 is 1.99 and $N(d_1)$ is 0.9767.

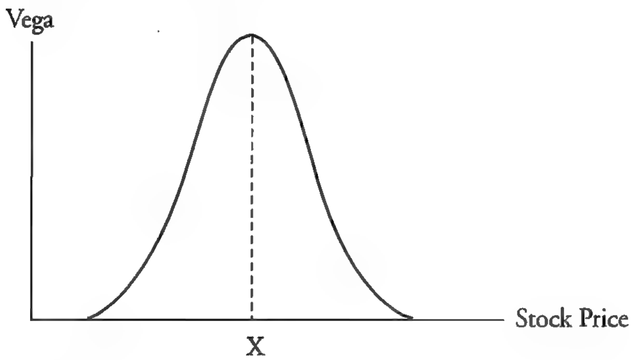
Answer:

$$\text{vega} = S_0 N'(d_1)\sqrt{T} = 50 \times 2.89 \times \sqrt{0.25} = 72.25$$

The interpretation for this value is that for a 1% increase in the volatility of the option (in this example, 12% to 13%), the value of the option will increase by approximately 0.01 × 72.25 = 0.7225.

Options are most sensitive to changes in volatility when they are at-the-money. Deep out-of-the-money or deep in-the-money options have little sensitivity to changes in volatility (i.e., vega is close to zero). The diagram in Figure 5 illustrates this behavior.

Figure 5: Vega of a Stock Option



Rho

Rho, ρ, measures an option's sensitivity to changes in the risk-free rate. Keep in mind, however, that equity options are not as sensitive to changes in interest rates as they are to changes in the other variables (e.g., volatility and stock price). Large changes in rates have only small effects on equity option prices. Rho is a much more important risk factor for fixed-income derivatives.

Rho for a call option is calculated using the following equation:

$$\text{rho} = \frac{\partial c}{\partial r}$$

where:
∂c = change in the call price
∂r = change in interest rate

In-the-money calls and puts are more sensitive to changes in rates than out-of-the-money options. Increases in rates cause larger *increases* for in-the-money call prices (versus out-of-the-money calls) and larger *decreases* for in-the-money puts (versus out-of-the-money puts).

For European calls on a non-dividend-paying stock, rho is measured as:

$$\text{rho(call)} = XTe^{-rT}\,N(d_2)$$

For European puts, rho is:

$$\text{rho(put)} = -XTe^{-rT}\,N(-d_2)$$

Example: Computing rho

Suppose that stock XYZ is trading at $50, and there is a call option that trades on XYZ with an exercise price of $45 which expires in three months. The risk-free rate is 5%, and the standard deviation of returns is 12% annualized. **Determine** the value of the call option's rho. Assume d_2 is 1.93 and $N(d_2)$ is 0.9732.

Answer:

$$\text{rho(call)} = 45 \times 0.25 \times e^{-0.05\times0.25} \times 0.9732 = 10.81$$

Similar to the interpretation of vega, a 1% increase in the risk-free rate (from 5% to 6%) will increase the value of the call option by approximately 0.01 × 10.81 = 0.1081.

Hedging in Practice

AIM 42.10: Describe how hedging activities take place in practice, and discuss how scenario analysis can be used to formulate expected gains and losses with option positions.

One of the main problems facing options traders is the expense associated with trying to maintain positions that are neutral to the Greeks. Although delta-neutral positions can be created, it is not as easy to find securities at reasonable prices that can mitigate the negative effects associated with gamma and vega.

To make things somewhat more manageable, large financial institutions usually adjust to a delta-neutral position and then monitor exposure to the other Greeks. Two offsetting situations assist in this monitoring activity. First, institutions that have sold options to their clients are exposed to negative gamma and vega, which tend to become more negative as time passes. In contrast, when the options are initially sold at-the-money, the level of sensitivity to gamma and vega is highest, but as time passes, the options tend to go

either in-the-money or out-of-the-money. The farther in- or out-of-the-money an option becomes, the less the impact of gamma and vega on the delta-neutral position.

Scenario analysis involves calculating expected portfolio gains or losses over desired periods using different inputs for underlying asset price and volatility. In this way, traders can assess the impact of changing various factors individually, or simultaneously, on their overall position.

Portfolio Insurance

AIM 42.11: Describe how portfolio insurance can be created through option instruments and stock index futures.

Portfolio insurance is the combination of (1) an underlying instrument and (2) either cash or a derivative that generates a floor value for the portfolio in the event that market values decline, while still allowing for upside potential in the event that market values rise.

The simplest way to create portfolio insurance is to buy put options on an underlying portfolio. In this case, any loss on the portfolio may be offset with gains on the long put position.

Simply buying puts on the underlying portfolio may not be feasible because the put options needed to generate the desired level of portfolio insurance may not be available. As an alternative to buying the puts, a synthetic put position can be created with index futures contracts. This is accomplished by selling index futures contracts in an amount equivalent to the proportion of the portfolio dictated by the delta of the required put option. The main reasons traders may prefer synthetically creating the portfolio insurance position with index futures include substantially lower trading costs and relatively higher levels of liquidity.

Key Concepts

1. A naked call option is written without owning the underlying asset, whereas a covered call is a short call option where the writer owns the underlying asset. Neither of these positions is a hedged position.

2. Stop-loss trading strategies are designed to minimize losses in the event the price of the underlying exceeds the strike price of a short call-option position.

3. Delta-neutral hedges are sophisticated hedging methods that minimize changes in a portfolio's position due to changes in the underlying security.

4. Delta-neutral hedges are only appropriate for small changes in the underlying asset and need to be rebalanced when large changes in the asset's value occur.

5. The delta of a portfolio is a weighted average of the deltas of each portfolio position.

6. Theta, also referred to as the time decay of an option, measures the sensitivity of an option's price to decreases in time to expiration.

7. Gamma measures the sensitivity of an option's price to changes in the option's delta.

8. Gamma is used to correct the hedging error associated with delta-neutral positions by providing added protection against large movements in the underlying asset's price.

9. Gamma-neutral positions are created by matching the gamma of the portfolio with an offsetting option position.

10. Theta, delta, and gamma directly affect the rate of return of an option portfolio.

11. Vega measures the sensitivity of an option's price to changes in the underlying asset's volatility.

12. Rho measures the sensitivity of an option's price to changes in the level of interest rates.

13. Hedging usually entails actively managing a delta-neutral position while monitoring the other option Greek sensitivities.

14. Portfolio insurance is the combination of (1) an underlying instrument and (2) either cash or a derivative that generates a floor value of the portfolio in the event that market valuations decline, while allowing for upside potential in the event that market valuations rise.

CONCEPT CHECKERS

1. Which of the following choices will effectively hedge a short call option position that exhibits a delta of 0.5?
   A. Sell two shares of the underlying for each option sold.
   B. Buy two shares of the underlying for each option sold.
   C. Sell the number of shares of the underlying equal to one-half the options sold.
   D. Buy the number of shares of the underlying equal to one-half the options sold.

2. A delta-neutral position exhibits a gamma of –3,200. An existing option with a delta equal to 0.5 exhibits a gamma of 1.5. Which of the following will generate a gamma-neutral position for the existing portfolio?
   A. Buy 4,800 of the available options.
   B. Sell 4,800 of the available options.
   C. Buy 2,133 of the available options.
   D. Sell 2,133 of the available options.

3. Which of the following actions would have to be taken to restore a delta-neutral hedge to the gamma-neutral position created in Question 2?
   A. Buy 1,067 shares of the underlying stock.
   B. Sell 1,067 shares of the underlying stock.
   C. Buy 4,266 shares of the underlying stock.
   D. Sell 4,266 shares of the underlying stock.

4. Portfolio insurance payoffs would not involve which of the following?
   A. Selling call options in the proportion 1/delta.
   B. Buying put options one-to-one relative to the underlying.
   C. Buying and selling the underlying in the proportion of delta of a put.
   D. Buying and selling futures in the proportion of delta of a put.

5. Which of the following statements about the "Greeks" is true?
   A. Rho for fixed income options is small.
   B. Call option deltas range from –1 to +1.
   C. A vega of 10 suggests that for a 1% increase in volatility, the option price will increase by 0.10.
   D. Theta is the most negative for out-of-the-money options.

Concept Checker Answers

1. **D** In order to hedge a short call option position, a manager would have to buy enough of the underlying to equal the delta times the number of options sold. In this case, delta = 0.5, so for every two options sold, the manager would have to buy a share of the underlying security.

2. **C** To create a gamma-neutral position, a manager must add the appropriate number of options that equals the existing portfolio gamma position. In this case, the existing gamma position is –3,200, and an available option exhibits a gamma of 1.5, which translates into buying approximately 2,133 options (3,200 / 1.5).

3. **B** The gamma-neutral hedge requires the purchase of 2,133 options, which will then increase the delta of the portfolio to 1,067 (2,133 × 0.5). Therefore, this would require selling approximately 1,067 shares to maintain a delta-neutral position.

4. **A** Portfolio insurance can be created by all of the statements except selling call options in the proportion 1/delta. This action generates a delta-neutral hedge, not portfolio insurance.

5. **C** Theta is the most negative for at-the-money options. Call option deltas range from 0 to 1. A vega of 10 suggests that for a 1% increase in volatility, the option price will increase by 0.10. Rho for equity options is small.

The following is a review of the Valuation and Risk Models principles designed to address the AIM statements set forth by GARP®. This topic is also covered in:

MEASURES OF FINANCIAL RISK

Topic 43

EXAM FOCUS

The assumption regarding the shape of the underlying return distribution is critical in determining an appropriate risk measure. The mean-variance framework can only be applied under the assumption of an elliptical distribution such as the normal distribution. The value at risk (VaR) measure can calculate risk measures when the return distribution is non-elliptical, but the measurement is unreliable and no estimate of the amount of loss is provided. Expected shortfall is a more robust risk measure that satisfies all the properties of a coherent risk measure with less restrictive assumptions. There is a lot of overlap between the material in this topic and material from Books 1 and 2 related to the efficient frontier, VaR, and skewness and kurtosis. For the exam, focus most of your attention on the properties of coherent risk measures and the expected shortfall methodology.

MEAN-VARIANCE FRAMEWORK

AIM 43.1: Describe the mean-variance framework and the efficient frontier.

The traditional mean-variance model estimates the amount of financial risk for portfolios in terms of the portfolio's expected return (i.e., mean) and risk (i.e., standard deviation or variance). Under the **mean-variance framework**, it is necessary to assume that return distributions for portfolios are elliptical distributions. The most commonly known elliptical probability distribution function is the normal distribution.

The **normal distribution** is a continuous distribution that illustrates all possible outcomes for random variables. If returns are normally distributed, approximately 66.7% of returns will occur within plus or minus one standard deviation of the mean. Approximately 95% of the observations will occur within plus or minus two standard deviations of the mean. Thus, given this type of distribution, returns are more likely to occur closer to the mean return. The probability density function for the normal distribution is shown as follows. Recall that the standard normal distribution has a mean of zero and a standard deviation of one.

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right]$$

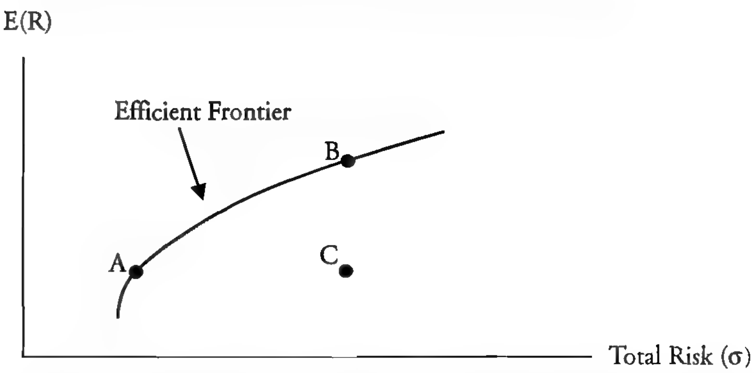
Portfolio managers are concerned with measuring downside risk and therefore are particularly interested in measuring the possibility of outcomes to the left or below the expected mean return. If the return distribution is symmetrical (like the normal distribution), then the standard deviation is an appropriate measure of risk when determining the probability that an undesirable outcome will occur.

If we assume that return distributions for all risky securities are normally distributed, then we can choose portfolios based on the expected returns and standard deviations of all possible combinations of risky securities. Figure 1 below illustrates the concept of the **efficient frontier.**

In theory, all investors prefer securities or portfolios that lie on the efficient frontier. Consider portfolios A, B, and C in Figure 1. If you had to choose between portfolios A and C, which one would you prefer and why? Since portfolios A and C have the same expected return, a risk-averse investor would choose the portfolio with the least amount of risk (which would be Portfolio A). Now if you had to choose between portfolios B and C, which one would you choose and why? Because portfolios B and C have the same amount of risk, a risk-averse investor would choose the portfolio with the higher expected return (which would be Portfolio B). We say that Portfolio B dominates Portfolio C with respect to expected return, and that Portfolio A dominates Portfolio C with respect to risk. Likewise, all portfolios on the efficient frontier dominate all other portfolios in the investment universe of risky assets with respect to either risk, return, or both.

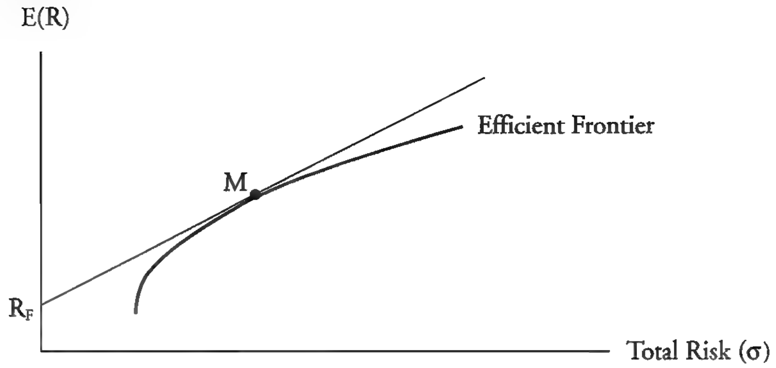
There are an almost unlimited number of combinations of risky assets to the right and below the efficient frontier. However, in the absence of a risk-free security, portfolios to the left and above the efficient frontier are not possible. Therefore, all investors will choose some portfolio on the efficient frontier. If an investor is more risk-averse, she may choose a portfolio on the efficient frontier closer to Portfolio A. If an investor is less risk-averse, she will choose a portfolio on the efficient frontier closer to Portfolio B.

Figure 1: The Efficient Frontier



If we now assume that there is a risk-free security, then the mean-variance framework is extended beyond the efficient frontier. Figure 2 illustrates that the optimal set of portfolios now lie on a straight line that runs from the risk-free security through the **market portfolio**, *M*. All investors will now seek investments by holding some portion of the risk-free security and the market portfolio. To achieve points on the line to the right of the market portfolio, an investor who is very aggressive will borrow money (at the risk-free rate) and invest in more of the market portfolio. More risk-averse investors will hold some combination of the risk-free security and the market portfolio to achieve portfolios on the line segment between the risk-free security and the market portfolio.

Figure 2: The Efficient Frontier with the Risk-Free Security



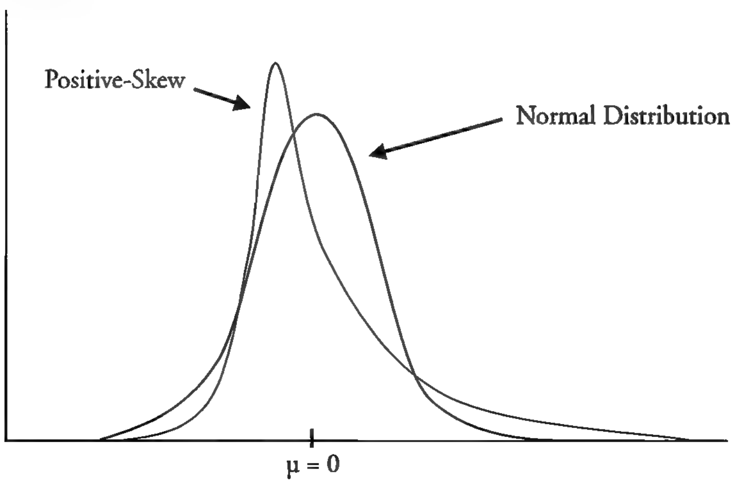
Mean-Variance Framework Limitations

AIM 43.2: Explain the limitations of the mean-variance framework with respect to assumptions about the return distributions.

The use of the standard deviation as a risk measurement is not appropriate for non-normal distributions. If the shape of the underlying return density function is not symmetrical, then the standard deviation does not capture the appropriate probability of obtaining undesirable return outcomes.

Figure 3 illustrates two probability distribution functions. One probability distribution function is the normal distribution with a mean of zero. The other probability distribution is positively skewed. This positively skewed distribution has the same mean and standard deviation as the normal distribution. The degree of skewness alters the entire distribution. For the positively skewed distribution, outcomes below the mean are more likely to occur closer to the mean. Clearly normality is an important assumption when using the mean-variance framework. Thus, the mean-variance framework is unreliable when the assumption of normality is not met.

Figure 3: Normal Distribution vs. Positively-Skewed Distribution



Value at Risk

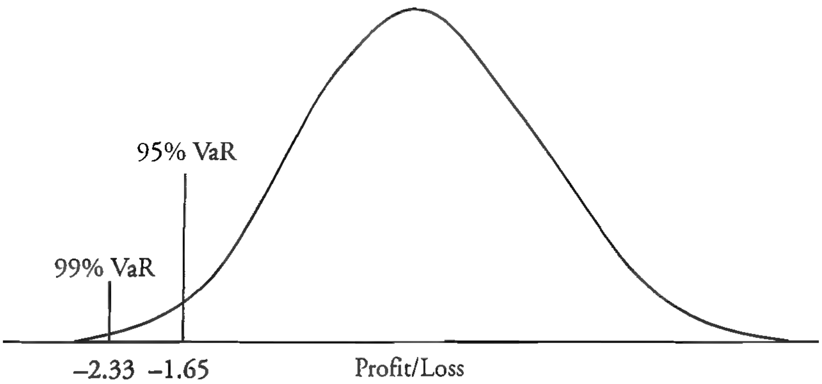
AIM 43.3: Define the value at risk (VaR) measure of risk, discuss assumptions about return distributions and holding period, and explain the limitations of VaR.

Value at risk (VaR) is interpreted as the worst possible loss under normal conditions over a specified period. Another way to define VaR is as an estimate of the maximum loss that can occur with a given confidence level. If an analyst says, "for a given month, the VaR is $1 million at a 95% level of confidence," then this translates to mean "under normal conditions, in 95% of the months (19 out of 20 months), we expect the fund to either earn a profit or lose no more than $1 million." Analysts may also use other standard confidence levels (e.g., 90% and 99%). Recall from Book 2 that delta-normal VaR can be computed using the following expression: $[\mu - (z)(\sigma)]$.

A major limitation of the VaR measure for risk is that two arbitrary parameters are used in the calculation—the confidence level and the holding period. The confidence level indicates the likelihood or probability that we will obtain a value greater than or equal to VaR. The holding period can be any pre-determined time period measured in days, weeks, months, or years.

Figure 4 illustrates VaR parameters for a confidence level of 95% and 99%. As you can see, the level of risk is dependent on the degree of confidence chosen. VaR increases when the confidence level increases. In addition, VaR will increase at an increasing rate as the confidence level increases.

Figure 4: VaR Measurements for a Normal Distribution



The second arbitrary parameter is the holding period. VaR will increase with increases in the holding period. The rate at which VaR increases is determined in part by the mean of the distribution. If the return distribution has a mean, μ, equal to 0, then VaR rises with the square root of the holding period (i.e., the square root of time) as was discussed in Book 2. If the return distribution has a $\mu > 0$, then VaR rises at a lower rate and will eventually decrease. Thus, the mean of the distribution is an important determinant for estimating how VaR changes with changes in the holding period.

VaR estimates are also subject to both model risk and implementation risk. Model risk is the risk of errors resulting from incorrect assumptions used in the model. Implementation risk is the risk of errors resulting from the implementation of the model.

Another major limitation of the VaR measure is that it does not tell the investor the amount or magnitude of the actual loss. VaR only provides the maximum value we can lose for a given confidence level. Two different return distributions may have the same VaR, but very different risk exposures. A practical example of how this can be a serious problem is when a portfolio manager is selling out-of-the-money options. For a majority of the time, the options will have a positive return and, therefore, the expected return is positive. However, in the unfavorable event that the options expire in-the-money, the resulting loss can be very large. Thus, different strategies focusing on lowering VaR can be very misleading since the magnitude of the loss is not calculated.

To summarize, VaR measurements work well with elliptical return distributions, such as the normal distribution. VaR is also able to calculate the risk for non-normal distributions; however, VaR estimates may be unreliable in this case. Limitations in implementing the VaR model for determining risk result from the underlying return distribution, arbitrary confidence level, arbitrary holding period, and the inability to calculate the magnitude of losses. The measure of VaR also violates the coherent risk measure property of subadditivity when the return distribution is not elliptical. This property is further explained in the next AIM.

Coherent Risk Measures

AIM 43.4: Define the properties of a coherent risk measure and explain the meaning of each property:

- **Explain why VaR is not a coherent risk measure.**

In order to properly measure risk, one must first clearly define what is meant by a measure of risk. If we allow *R* to be a set of random events and $\rho(R)$ to be the risk measure for the random events, then **coherent risk measures** should exhibit the following properties:

1. **Monotonicity**: a portfolio with greater future returns will likely have less risk: $R_1 \geq R_2$, then $\rho(R_1) \leq \rho(R_2)$

2. **Subadditivity**: the risk of a portfolio is at most equal to the risk of the assets within the portfolio: $\rho(R_1 + R_2) \leq \rho(R_1) + \rho(R_2)$

3. **Positive homogeneity**: the size of a portfolio, β, will impact the size of its risk: for all $\beta > 0$, $\rho(\beta R) = \beta\rho(R)$

4. **Translation invariance**: the risk of a portfolio is dependent on the assets within the portfolio: for all constants *c*, $\rho(c + R) = \rho(R) - c$

The first, third, and fourth properties are more straightforward properties of well-behaved distributions. Monotonicity infers that if a random future value R_1 is always greater than a random future value R_2, then the risk of the return distribution for R_1 is less than the risk of the return distribution for R_2. Positive homogeneity suggests that the risk of a position

is proportional to its size. Positive homogeneity should hold as long as the security is in a liquid market. Translation invariance implies that the addition of a sure amount reduces the risk at the same rate as the cash needed to make the position acceptable.

Subadditivity is the most important property for a coherent risk measure. The property of subadditivity states that a portfolio made up of sub-portfolios will have equal or less risk than the sum of the risks of each individual sub-portfolio. This assumes that when individual risks are combined, there may be some diversification benefits or, in the worst case, no diversification benefits and no greater risk. This implies grouping or adding risks does not increase the overall aggregate risk amount.

Expected Shortfall

AIM 43.5: Explain and calculate expected shortfall (ES), and compare and contrast VaR and ES.

Value at risk is the minimum percent loss, equal to a pre-specified worst case quantile return (typically the 5th percentile return). **Expected shortfall** (ES) is the expected loss given that the portfolio return already lies below the pre-specified worst case quantile return (i.e., below the 5th percentile return). In other words, expected shortfall is the mean percent loss among the returns falling below the q-quantile. Expected shortfall is also known as **conditional VaR** or **expected tail loss** (ETL).

For example, assume an investor is interested in knowing the 5% VaR (the 5% VaR is equivalent to the 5th percentile return) for a fund. Further, assume the 5th percentile return for the fund equals –20%. Therefore, 5% of the time, the fund earns a return less than –20%. The value at risk is –20%. However, VaR does not provide good information regarding the expected size of the loss if the fund performs in the lower 5% of the possible outcomes. That question is answered by the expected shortfall amount, which is the expected value of all returns falling below the 5th percentile return (i.e., below –20%). Therefore, expected shortfall will equal a larger loss than the VaR. In addition, unlike VaR, ES has the ability to satisfy the property of subadditivity.

The ES method provides an estimate of how large of a loss is expected if an unfavorable event occurs. VaR did not provide any estimate of the magnitude of losses, only the probability that they might occur. The property of subadditivity under the ES framework is also beneficial in eliminating another problem for VaR. When adjusting both the holding period and confidence level at the same time, an ES surface curve showing the interactions of both adjustments is convex. This implies that the ES method is more appropriate than the VaR method in solving portfolio optimization problems.

ES is similar to VaR in that both provide a common consistent risk measure across different positions. ES can be implemented in determining the probability of losses the same way that VaR is implemented as a risk measure, and they both appropriately account for correlations.

However, ES is a more appropriate risk measure than VaR for the following reasons:

- ES satisfies all of the properties of coherent risk measurements including subadditivity. VaR only satisfies these properties for normal distributions.
- The portfolio risk surface for ES is convex because the property of subadditivity is met. Thus, ES is more appropriate for solving portfolio optimization problems than the VaR method.
- ES gives an estimate of the magnitude of a loss for unfavorable events. VaR provides no estimate of how large a loss may be.
- ES has less restrictive assumptions regarding risk/return decision rules.

AIM 43.6: Describe spectral risk measures, and explain how VaR and ES are special cases of spectral risk measures.

A more general risk measure than either VaR or ES is known as the **risk spectrum** or risk aversion function. The risk spectrum measures the weighted averages of the return quantiles from the loss distributions. ES is a special case of this risk spectrum measure. When modeling the ES case, the weighting function is set to [1 / (1 – confidence level)] for tail losses. All other quantiles will have a weight of zero.

VaR is also a special case of spectral risk measure models. The weighting function with VaR assigns a probability of one to the event that the p-value equals the level of significance (i.e., $p = \alpha$), and a probability of zero to all other events where $p \neq \alpha$. Thus, the ES measure places equal weights on tail losses while VaR places no weight on tail losses.

In order for a risk measure to be coherent, it must give higher losses at least the same weight as lower losses. In the ES case, all losses are given the same weight. This suggests that investors are risk-neutral with respect to losses. This is contradictory to the common notion that investors are risk-averse. In the VaR case, only the loss associated with a p-value equal to α is given any weight. Greater losses are given no weight at all. This implies that investors are risk-seekers. Thus, the ES and VaR measures are inadequate in that the weighting function is not consistent with risk aversion.

Scenario Analysis

AIM 43.7: Describe how the results of scenario analysis can be interpreted as coherent risk measures.

The results of scenario analysis can be interpreted as coherent risk measures by first assigning probabilities to a set of loss outcomes. These losses can be thought of as tail drawings of the relevant distribution function. The expected shortfall for the distribution can then be computed by finding the arithmetic average of the losses. Therefore, the outcomes of scenario analysis must be coherent risk measurements, because ES is a coherent risk measurement.

Scenario analysis can also be applied in situations where there are numerous distribution functions involved. It can be shown that the ES, the highest ES from a set of comparable expected shortfalls based on different distribution functions, and the highest expected shortfall from a set of highest losses are all coherent risk measures. For example, assume you are considering a set of *n* loss outcomes out of a family of distribution functions. The

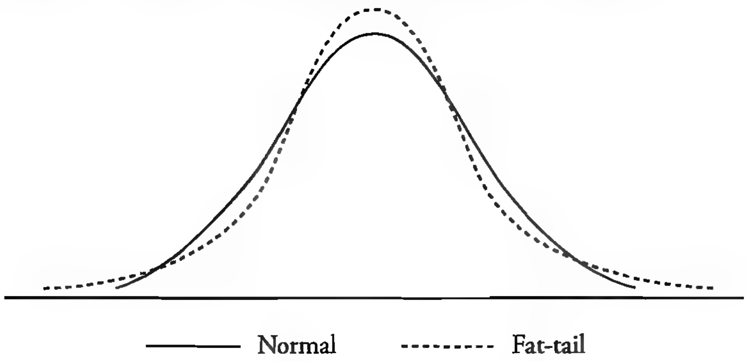
ES is obtained from each distribution function. If there is a set of *m* comparable expected shortfalls, that each have a different corresponding loss distribution function, then the maximum of these expected shortfalls is a coherent risk measure. Thus, in cases where n = 1, the ES is the same as the probable maximum loss because there is only one tail loss in each scenario. If *m* equals one, then the highest expected loss from a single scenario analysis is a coherent measure. In cases where *m* is greater than one, the highest expected of *m* worst case outcomes is a coherent risk measure.

Skewness and Kurtosis

Any statistical distribution can be described in terms of mean, standard deviation, skewness, and kurtosis. For normal distributions, the first two parameters or moments of a distribution, the mean and standard deviation, can be any values as long as the standard deviation is not negative. Skewness is zero, indicating the probability distribution function is symmetrical, and kurtosis is equal to three. In addition to measuring the middle of the distribution, skewness and kurtosis can be used to measure the shape of the tails.

Distributions with large skewness or kurtosis have a higher probability of observations occurring in the tails relative to the normal distribution. Distributions with high skewness or kurtosis are sometimes referred to as fat-tailed or heavy-tailed distributions. As illustrated in Figure 5, there is a larger probability of an observation occurring further away from the mean of the distribution. The first two moments (mean and variance) of the distributions are similar for the fat-tailed and normal distribution. However, in addition to the greater mass in the tails, there is also a greater probability mass around the mean for the fat-tailed distribution. Also, there is less probability mass in the intermediate range, around plus or minus one standard deviation, for the fat-tailed distribution compared to the normal distribution.

Figure 5: Heavy-tail Distribution vs. Normal Distribution



If the return distribution is not normally distributed, then standard deviation is not an appropriate risk measure. Figures 3 and 5 illustrate that with larger skewness and higher kurtosis, standard deviation does not capture the probability of returns occurring in the tail of the distribution. Tail heaviness is defined as a distribution with a kurtosis greater than three. The mean-variance framework underestimates the amount of risk when distributions have high kurtosis and skewness [in particular, left (or negative) skewness].

Key Concepts

1. The traditional mean-variance model estimates the amount of financial risk for portfolios in terms of the portfolio's expected return (mean) and risk (standard deviation or variance). A necessary assumption for this model is that return distributions for the portfolios are elliptical distributions. The efficient frontier is the set of portfolios that dominate all other portfolios in the investment universe of risky assets with respect to risk and return. When a risk-free security is introduced, the optimal set of portfolios consists of a line from the risk-free security that is tangent to the efficient frontier at the market portfolio.

2. The mean-variance framework is unreliable when the underlying return distribution is not normal or elliptical. The standard deviation is not an accurate measure of risk and does not capture the probability of obtaining undesirable return outcomes when the underlying return density function is not symmetrical.

3. Value at risk (VaR) is a risk measurement that determines the probability of an occurrence in the left-hand tail of a return distribution at a given confidence level. VaR is defined as: $[\mu - (z)(\sigma)]$. The underlying return distribution, arbitrary choice of confidence levels and holding periods, and the inability to calculate the magnitude of losses result in limitations in implementing the VaR model when determining risk.

4. The properties of a coherent risk measure are:
   - Monotonicity: $Y \geq X \Rightarrow \rho(Y) \leq \rho(X)$.
   - Subadditivity: $\rho(X + Y) \leq \rho(X) + \rho(Y)$.
   - Positive homogeneity: $\rho(hx) = h\rho(X)$ for $h > 0$.
   - Translation invariance: $\rho(X + n) = \rho(X) - n$.

   Subadditivity, the most important property for a coherent risk measure, states that a portfolio made up of sub-portfolios will have equal or less risk than the sum of the risks of each individual sub-portfolio.

5. Expected shortfall is a more accurate risk measure than VaR for the following reasons:
   - ES satisfies all the properties of coherent risk measurements including subadditivity.
   - The portfolio risk surface for ES is convex since the property of subadditivity is met. Thus, ES is more appropriate for solving portfolio optimization problems than the VaR method.
   - ES gives an estimate of the magnitude of a loss for unfavorable events. VaR provides no estimate of how large a loss may be.
   - ES has less restrictive assumptions regarding risk/return decision rules.

6. ES is a special case of the risk spectrum measure where the weighting function is set to 1 / (1 – confidence level) for tail losses that all have an equal weight, and all other quantiles have a weight of zero. The VaR is a special case where only a single quantile is measured, and the weighting function is set to one when p-value equals the level of significance, and all other quantiles have a weight of zero.

7. The outcomes of scenario analysis are coherent risk measurements, because expected shortfall is a coherent risk measurement. The ES for the distribution can be computed by finding the arithmetic average of the losses for various scenario loss outcomes.

8. Distributions with large skewness or kurtosis have a higher probability of observations occurring in the tails relative to the normal distribution. When return distributions have higher skewness or kurtosis, the use of standard deviation is not an appropriate risk measurement because it does not capture the probability of returns occurring in the left-tail of the distribution.

Concept Checkers

1. The mean-variance framework is inappropriate for measuring risk when the underlying return distribution:
   A. is normal.
   B. is elliptical.
   C. has a kurtosis equal to three.
   D. is positively skewed.

2. Assume an investor is very risk-averse and is creating a portfolio based on the mean-variance model and the risk-free asset. The investor will most likely choose an investment on the:
   A. left-hand side of the efficient frontier.
   B. right-hand side of the efficient frontier.
   C. line segment connecting the risk-free rate to the market portfolio.
   D. line segment extending to the right of the market portfolio.

3. $\rho(X + Y) \leq \rho(X) + \rho(Y)$ is the mathematical equation for which property of a coherent risk measure?
   A. Monotonicity.
   B. Subadditivity.
   C. Positive homogeneity.
   D. Translation invariance.

4. Which of the following is not a reason that expected shortfall (ES) is a more appropriate risk measure than value at risk (VaR)?
   A. For normal distributions, only ES satisfies all the properties of coherent risk measurements.
   B. For non-elliptical distributions, the portfolio risk surface formed by holding period and confidence level is more convex for ES.
   C. ES gives an estimate of the magnitude of a loss.
   D. ES has less restrictive assumptions regarding risk/return decision rules than VaR.

5. If the weighting function in the general risk spectrum measure is set to 1 / (1 – confidence level) for all tail losses, then the risk spectrum is a special case of:
   A. value at risk.
   B. mean-variance.
   C. expected shortfall.
   D. scenario analysis.

Concept Checker Answers

1. **D** The mean-variance framework is only appropriate when the underlying distribution is elliptical. The normal distribution is a special case of elliptical distributions where skewness is equal to zero and kurtosis is equal to three. If there is any skewness, the distribution function will not be symmetrical, and standard deviation will not be an appropriate risk measure.

2. **C** Under the mean-variance framework, when a risk-free security is included in the analysis, the optimal set of portfolios lies on a straight line that runs from the risk-free security to the market portfolio. All investors will hold some portion of the risk-free security and the market portfolio. More risk-averse investors will hold some combination of the risk-free security and the market portfolio to achieve portfolios on the line segment between the risk-free security and the market portfolio.

3. **B** The property of subadditivity states that a portfolio made up of sub-portfolios will have equal or less risk than the sum of the risks of each individual sub-portfolio.

4. **A** VaR and ES both satisfy all the properties of coherent risk measures for normal distributions. However, only ES satisfies all the properties of coherent risk measures when the assumption of normality is not met.

5. **C** Expected shortfall is a special case of the risk spectrum measure that is found by setting the weighting function to 1 / (1 – confidence level) for tail losses that all have an equal weight.

The following is a review of the Valuation and Risk Models principles designed to address the AIM statements set forth by GARP®. This topic is also covered in:

Putting VaR to Work

Topic 44

Exam Focus

Derivatives and portfolios containing derivatives and other assets create challenges for risk managers in measuring value at risk (VaR). In this topic, risk measurement approaches are discussed for linear and non-linear derivatives. The advantages and disadvantages and underlying assumptions of the various approaches are presented. In addition, Taylor Series approximation is addressed, with examples of applying this theory to VaR approaches. Finally, structured Monte Carlo (SMC), stress testing, and worst case scenario (WSC) analysis are presented as useful methods in extending VaR techniques to more appropriately measure risk for complex derivatives and scenarios.

Linear vs. Non-Linear Derivatives

AIM 44.1: Explain and give examples of linear and non-linear derivatives.

A derivative is described as *linear* when the relationship between an underlying factor and the derivative is linear in nature. For example, an equity index futures contract is a linear derivative, while an option on the same index is non-linear. The delta for a linear derivative must be constant for all levels of the underlying factor, but not necessarily equal to one.

For example, the rate on a foreign currency forward contract is defined as:

$$F_{t,T} = S_t(1+R_D)/(1+R_F)$$

Where $F_{t,T}$ is the forward rate at time t for the period T-t, S_t is the spot exchange rate, R_D is the domestic interest rate, and R_F is the foreign interest rate. The value at risk (VaR) of the forward is related to the spot rate, S_t, and the foreign and domestic interest rates. Assuming fixed interest rates for very short time intervals, we can approximate the forward rate, $F_{t,T}$, with the interest rate differential as a constant K that is not a function of time as follows:

$$F_{t,T} = S_t(1 + R_D)/(1 + R_F) \approx KS_t$$

Furthermore, the continuously compounded return on the foreign forward contract, $\Delta f_{t,t+1}$, is approximately equal to the return on the spot rate, $\Delta s_{t,t+1}$. This can be shown mathematically where the ln of the constant K is very close to zero and the approximate relationship is simplified as follows:

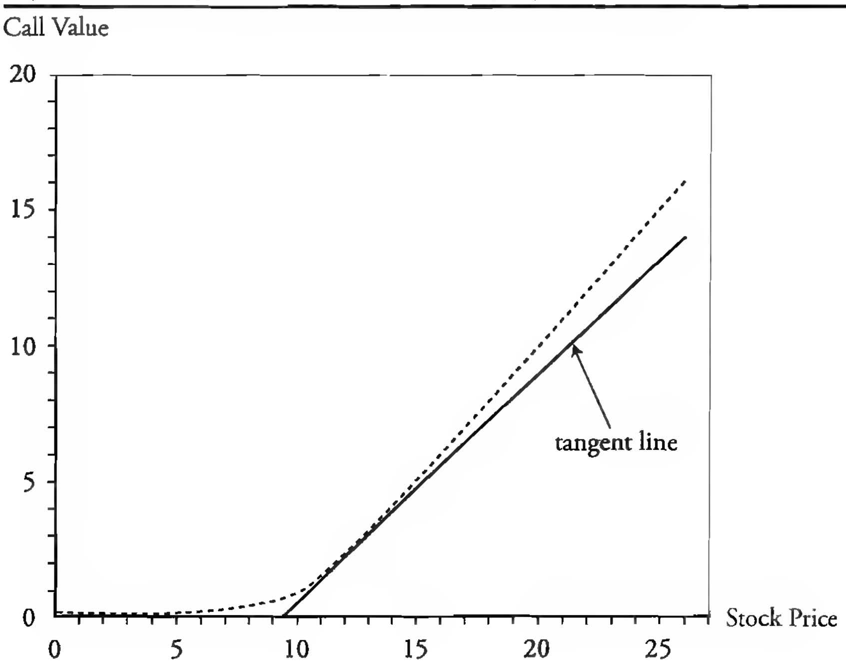
$$\Delta f_{t,t+1} = \ln(F_{t+1,T-1}/F_{t,T}) = \ln(S_{t+1}/S_t) + \ln(\Delta K) \approx \ln(S_{t+1}/S_t)$$

Changes in exchange rates can therefore be approximated by changes in spot rates. The VaR of a spot position is approximately equal to a forward position exchange rate if the only relevant underlying factor is the exchange rate. As this illustrates, many derivatives that are referred to as linear are actually only approximately linear. If we account for the changes in the two interest rates, the actual relationship would be nonlinear. Thus, the notion of linearity or nonlinearity is a function of the definition of the underlying risk factor.

The value of a ***nonlinear*** derivative is a function of the change in the value of the underlying asset and is dependent on the state of the underlying asset. A call option is a good example of a nonlinear derivative. The value of the call option does not increase (decrease) at a constant rate when the underlying asset increases (decreases) in value.

The change in the value of the call option is dependent in part on how far away the market value of the stock is from the exercise price. Thus, the relationship of the stock to the exercise price, *S/X*, captures the distance the option is from being in-the-money. Figure 1 illustrates how the value of the call option does not change at a constant rate with the change in the value of the underlying asset. The curved line represents the actual change in value of the call option based on the Black-Scholes-Merton model. The tangent line at any point on the curve illustrates how this is not a linear change in value. Furthermore, the slope of the line increases as the stock price increases. The percentage change in the call value given a change in the underlying stock price will be different for different stock price levels.

Figure 1: Call Option Value Given Underlying Stock Price



AIM 44.2: Explain how to calculate VaR for linear derivatives.

In general, the VaR of a long position in a linear derivative is $\text{VaR}_p = \Delta\text{VaR}_f$, where VaR_f is the VaR of the underlying factor and the derivative's delta, Δ, is the sensitivity of the derivative's price to changes in the underlying factor. Delta is assumed to be positive because we're modeling a long position. The local delta is defined as the percentage change in the derivative's price for a 1% change in the underlying asset. For small changes in the underlying price of the asset the change in price of the derivative can be extrapolated based on the local delta.

Example: Futures contract VaR

Determine how a risk manager could estimate the VaR of an equity index futures contract. Assume a 1-point increase in the index increases the value of a long position in the contract by $500.

Answer:

This relationship is shown mathematically as: $F_t = \$500S_t$, where F_t is the futures contract and S_t is the underlying index. The VaR of the futures contract is calculated as the amount of the index point movement in the underlying index, S_t, times the multiple, $500 as follows: $\text{VaR}(F_t) = \$500\text{VaR}(S_t)$.

Taylor Approximation

AIM 44.3: Describe the delta-normal approach to calculating VaR for non-linear derivatives.

AIM 44.4: Describe the limitations of the delta-normal method.

Suppose we create a table that shows the relationship of the call value to the stock price. The original stock price and call option value are $11.00 and $1.41, respectively. The Black-Scholes-Merton model is used to calculate the call value for different stock prices. Figure 2 summarizes some of the points.

Figure 2: Change in Call Value Given a Change in Stock Price (numbers reflect small rounding error)

Stock Price, S	$7.00	$8.00	$9.00	$10.00	$10.89	$11.00
Value of Call, C	$0.00	$0.05	$0.23	$0.69	$1.32	$1.41
Percentage Decrease in S	–36.36%	–27.27%	–18.18%	–9.09%	–1.00%	
Percentage Decrease in C	–100.00%	–96.76%	–83.31%	–51.06%	–6.35%	
Delta (ΔC% / ΔS%)	2.74	3.55	4.58	5.62	6.35	

The **delta** is calculated in Figure 2 by dividing the percentage change in the call value by the percentage change in the stock price (delta = $\Delta C\% / \Delta S\%$). The **local delta** is the slope of the line at any point of the nonlinear relationship for a 1% change in the stock price. The local delta can be used to estimate the change in the value of the call option given a *small* change in the value of the stock price.

Example: Call option VaR

Suppose a 6-month call option with a strike price, *X*, of $10 is currently trading for $1.41, when the market price of the underlying stock is $11. A 1% decrease in the stock price to $10.89 results in a 6.35% decrease in the call option with a value of $1.32. If the annual volatility of the stock is $\sigma = 0.1975$ and the risk-free rate of return is 5%, **calculate** the one day 5% VaR for this call option.

Answer:

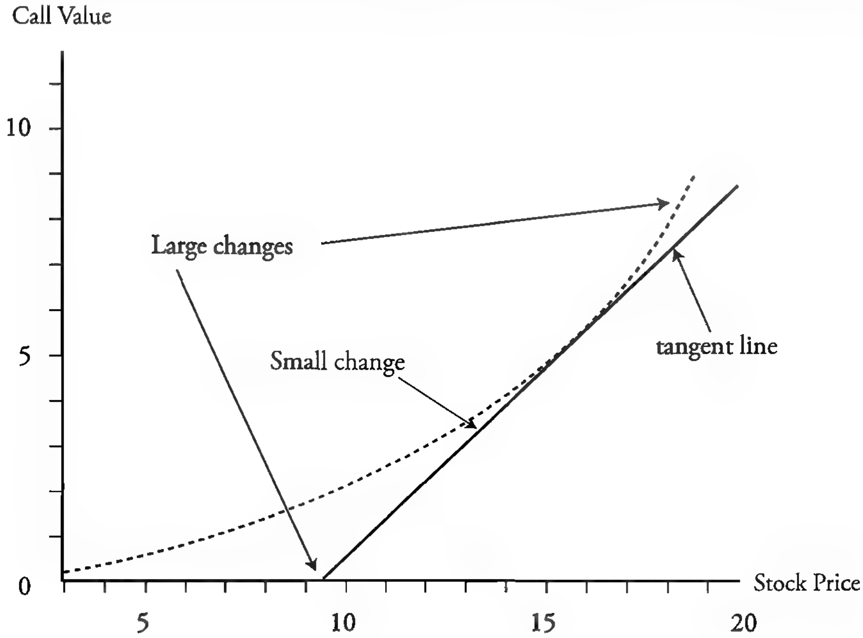
The daily volatility is approximately equal to 1.25% ($0.1975/\sqrt{250}$). The 5% VaR for the stock price is equivalent to a one standard deviation move, or 1.65 for the normal curve. Assuming a random walk or 0 mean daily return, the 5% VaR of the underlying stock is 0 – 1.25%(1.65) = –2.06%. A 1% change in the stock price results in a 6.35% change in the call option value, therefore, the delta = 0.0635/0.01 = 6.35. For small moves, delta can be used to estimate the change in the derivative given the VaR for the underlying asset as follows: $\text{VaR}_{\text{call}} = \Delta \text{VaR}_{\text{stock}}$ = 6.35(2.06%) = 0.1308 or 13.1%. In words, the 5% VaR implies there is a 5% probability that the call option value will decline by 13.1% or more. Note this estimate is only an approximation for small changes in the underlying stock. The precise change can be calculated using the Black-Scholes-Merton model.

Figure 3 illustrates that the slope of the line is only useful in estimating the call value with small changes in the underlying stock value. The gap between the tangency line representing the delta or slope of the line at the tangency point widens the further away the estimate is from the point of tangency. The first derivative of a function tells us the slope of the line at any given point. The second derivative tells us the rate of change. This information is summarized mathematically in the Taylor Series **approximation** of the function f(x) as follows:

$$f(x) \approx f(x_0) + f'(x_0)(x - x_0) + \frac{1}{2}f''(x_0)(x - x_0)^2$$

The Taylor Series states that the change in value of any function can be expressed by adjusting the original function value, $f(x_0)$ plus the slope of the line, $f'(x_0)$, times the change in the *x* variable plus the rate of the change measured by the last term above. The last term captures the convexity or curvature. This is still an approximation, but it is much closer than the linear estimation.

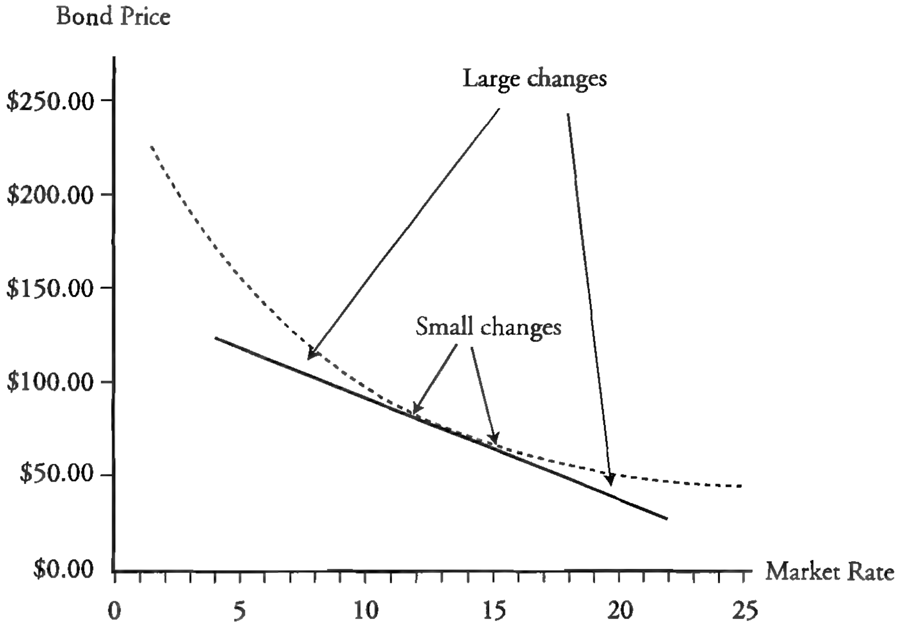
Figure 3: Call Option Example of Measurement Error Resulting from Convexity



Bond Example

The **price-yield (P-Y) curve** depicts the change in the value of a bond as market rates of interest change. This is another example of a nonlinear relationship. Figure 4 illustrates the P-Y curve for a 20-year treasury bond with no embedded options. The straight line represents the duration of the bond. Duration is a linear estimation of the change in bond price given a change in interest rates and is only good for very small changes. Conversely, for large changes in interest rates, the gap between the P-Y curve and the tangent line represents the estimation error. Measuring the convexity in addition to the duration of the bond gives a much better approximation of the change in bond price for a given change in market rates. The use of duration and convexity to estimate bond prices as interest rates change is similar to the use of the delta and of the gamma approximation of the impact of fluctuations in the underlying factor on the value of an option. Both approximations are based on the Taylor Series that uses first and second derivatives of a known pricing model.

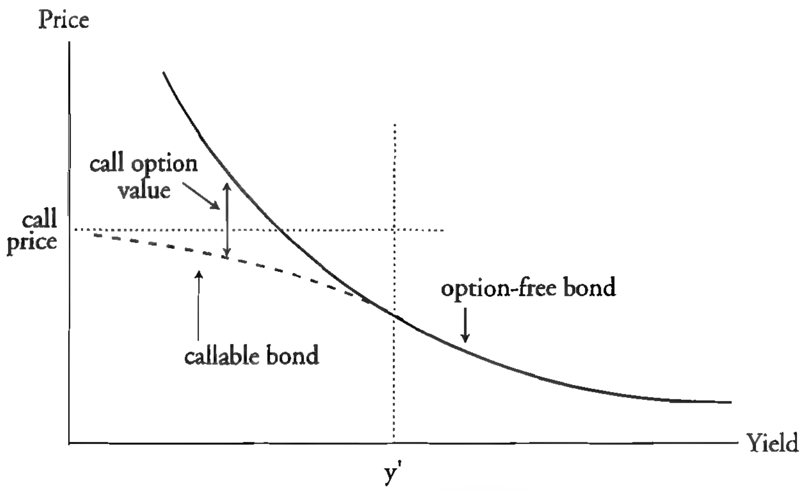
Figure 4: Measurement Error Resulting from Convexity in Bond Pricing



Consider a bond that is callable. The Price-Yield curve in Figure 5 illustrates that the call feature causes the P-Y curve to become concave as market interest rates approach the level where the bond will be called. Thus, the Taylor approximation is not useful because the callable bond is not a "well-behaved" function. In other words, the embedded call option causes the P-Y curve to deviate from the quadratic function that can be approximated by a polynomial of order two using the Taylor series.

Another example of a security with an embedded option are mortgage-backed securities (MBS). Borrowers will prepay loans early with significant drops in market interest rates. This causes the MBS to act similar to a bond that is called in. Unpredictable changes in duration due to early payoffs of MBS make the securities difficult to price and hedge. A convexity adjustment alone is not sufficient to estimate the change in the underlying security's value based on changes in market rates. The function explaining the relationship between the MBS value and market rates of interest does not behave similar at low and high levels of interest rates.

Figure 5: Price-Yield Curves for Callable and Noncallable Bonds



The Delta-Normal and Full Revaluation Methods

AIM 44.5: Explain the full revaluation method for computing VaR.

AIM 44.6: Compare delta-normal and full revaluation approaches.

Both the delta-normal and full-revaluation methods measure the risk of nonlinear securities. The **full-revaluation approach** calculates the VaR of the derivative by valuing the derivative based on the underlying value of the index after the decline corresponding to an *x*% VaR of the index. This approach is accurate, but can be highly computational. The revaluation of portfolios that include more complex derivatives (i.e., mortgage backed securities, or options with embedded features) are not easily calculated due to the large number of possible scenarios.

The **delta-normal approach** calculates the risk using the delta approximation ($VaR_p = \Delta VaR_f$), which is linear or the delta-gamma approximation, $f(x) \approx f(x_0) + f'(x_0)(x - x_0) + \frac{1}{2}f''(x_0)(x - x_0)^2$, which adjusts for the curvature of the underlying relationship. This approach simplifies the calculation of more complex securities by approximating the changes based on linear relationships (delta).

The Monte Carlo Approach

AIM 44.7: Explain structural Monte Carlo, stress testing and scenario analysis methods for computing VaR, identifying strengths and weaknesses of each approach.

AIM 44.8: Discuss the implications of correlation breakdown for scenario analysis.

The **structured Monte Carlo (SMC) approach** simulates thousands of valuation outcomes for the underlying assets based on the assumption of normality. The VaR for the portfolio of derivatives is then calculated from the simulated outcomes. The general equation assumes the underlying asset has normally distributed returns with a mean of μ and a standard deviation of σ. An example of a simulation equation is as follows:

$$s_{t+1,i} = s_t e^{\mu + \sigma \times z}$$

where $s_{t+1,i}$ is the simulated value for a continuously compounded return, based on a random draw, z_i, from a normal distribution with the given first and second moments. Therefore, the draws from the normal distribution are denoted by z_1, z_2, z_3,... z_N, and the N scenarios are $\mu + \sigma z_1$, $\mu + \sigma z_2$, $\mu + \sigma z_3$, ..., $\mu + \sigma z_N$. The N outcomes are then ordered and the $(1 - x/100) \times$ Nth value is the x% value.

Professor's Note: Notice how the previous simulation equation is very similar to the approximation to geometric Brownian motion (GBM) discussed in the Monte Carlo Methods topic in Book 2. The only difference is the use of a continuously compounded return in this example.

An *advantage* of the SMC approach is that it is able to address multiple risk factors by assuming an underlying distribution and modeling the correlations among the risk factors. For example, a risk manager can simulate 10,000 outcomes and then determine the probability of a specific event occurring. In order to run the simulations, the risk manager just needs to provide parameters for the mean and standard deviation and assume all possible outcomes are normally distributed.

A *disadvantage* of the SMC approach is that in some cases it may not produce an accurate forecast of future volatility and increasing the number of simulations will not improve the forecast.

Example: SMC approach

Suppose a risk manager requires a VaR measurement of a long straddle position. **Demonstrate** how a SMC approach will be implemented to estimate the VaR for a long straddle position.

Answer:

The straddle represents a portfolio of a long call and long put that anticipates a large movement up or down in the underlying stock. The typical VaR measurement would require an estimate of the underlying stock moving more than one standard deviation. However, in a straddle position, the VaR occurs when the stock does not move in price or only moves a small amount. The SMC approach simulates thousands of possible movements in the underlying stock and then uses those outcomes to estimate the VaR for the straddle position.

Correlations During Crisis

The key point here is that in times of crisis, correlations increase (some substantially) and strategies that rely on low correlations fall apart in those times. Certain economic or crisis events can cause diversification benefits to deteriorate in times when the benefits are most needed. A contagion effect occurs with a rise in volatility and correlation causing a different return generating process. Some specific examples of events leading to the breakdown of historical correlation matrices are the Asian crisis, the U.S. stock market crash of October 1987, the events surrounding the failure of Long-Term Capital Management (LTCM), and the recent global credit crisis.

A simulation using the SMC approach is not capable of predicting scenarios during times of crisis if the covariance matrix was estimated during normal times. Unfortunately, increasing the number of simulations does not improve predictability in any way.

For example, the probability of a four or more standard deviation event occurring based on the normal curve is 6.4 out of 100,000 times. However, suppose the number of times the daily return for the equity index is four or more standard deviations based on historical returns is approximately 500 out of 100,000 times. Based on the historical data a four or more standard deviation event is expected to occur once every 0.8 years, not once every 62 years implied by the normal curve.

Stress Testing

During times of crisis, a contagion effect often occurs where volatility and correlations both increase, thus mitigating any diversification benefits. *Stressing* the correlation is a method used to model the contagion effect that could occur in a crisis event.

One approach for stress testing is to *examine historical crisis* events, such as the Asian crisis, October 1987 market crash, etc. After the crisis is identified, the impact on the current portfolio is determined. The *advantage* of this approach is that no assumptions of underlying asset returns or normality are needed. The biggest *disadvantage* of using historical events for stress testing is that it is limited to only evaluating events that have actually occurred.

The **historical simulation approach** does not limit the analysis to specific events. Under this approach, the entire data sample is used to identify "extreme stress" situations for different asset classes. For example, certain historical events may impact the stock market more than the bond market. The objective is to identify the five to ten worst weeks for specific asset classes and then evaluate the impact on today's portfolio. The *advantage* of this approach is that it may identify a crisis event that was previously overlooked for a specific asset class. The focus is on identifying extreme changes in valuation instead of extreme movements in underlying risk factors. The *disadvantage* of the historical simulation approach is that it is still limited to actual historical data.

An alternative approach is to analyze different predetermined *stress scenarios*. For example, a financial institution could evaluate a 200bp increase in short-term rates, an extreme inversion of the yield curve or an increase in volatility for the stock market. As in the previous method, the next step is then to evaluate the effect of the stress scenarios on the current portfolio.

An *advantage* to scenario analysis is that it is not limited to the evaluation of risks that have occurred historically. It can be used to address any possible scenarios. A *disadvantage* of the stress scenario approach is that the risk measure is deceptive for various reasons. For example, a shift in the domestic yield curve could cause estimation errors by overstating the risk for a long and short position and understating the risk for a long-only position. Asset-class-specific risk is another disadvantage of the stress scenario approach. For example, emerging market debt, mortgage-backed securities, and bonds with embedded options all have unique asset class specific features such that interest rate risk only explains a portion of total risk. Addressing asset class risks is even more crucial for financial institutions specializing in certain products or asset classes.

Worst Case Scenario Measure

AIM 44.9: Describe worst case scenario analysis.

The **worst case scenario** (WCS) assumes that an unfavorable event will occur with certainty. The focus is on the distribution of worst possible outcomes given an unfavorable event. An expected loss is then determined from this worst case distribution analysis. Thus, the WCS information extends the VaR analysis by estimating the extent of the loss given an unfavorable event occurs.

In other words, the tail of the original return distribution is more thoroughly examined with another distribution that includes only probable extreme events. For example, within the lowest 5% of returns, another distribution can be formed with just those returns and a 1% WCS return can then be determined. Recall that VaR provides a value of the minimum loss for a given percentage, but says nothing about the severity of the losses in the tail. WCS analysis attempts to complement the VaR measure with analysis of returns in the tail.

Example: WCS approach

Suppose a risk manager simulates the data in Figure 6 using 10,000 random vectors for time horizons, H, of 20 and 100 periods. **Demonstrate** how a risk manager would interpret results for the 1% VaR and 1% WCS for a 100 period horizon.

Figure 6: Simulated Worst Case Scenario (WCS) Distribution

Time Horizon = H	*H = 20*	*H = 100*
Expected number of $Z_i < -2.33$	0.40	1.00
Expected number of $Z_i < -1.65$	1.00	4.00
Expected WCS	–1.92	–2.74
WCS 1 percentile	–3.34	–3.85
WCS 5 percentile	–2.69	–3.17

Answer:

Based on the simulation results in Figure 6, the 1% VaR assuming normality corresponds to –2.33 and over the next 100 trading periods a return worse than –2.33 is expected to occur one time. The 1% worst case scenario, denoted in this example by Z_i is –3.85. Thus, over the next 100 trading periods a return worse than –2.33 is expected to occur one time. More importantly, the size of that return is expected to be –2.74, with a 1% probability that the return will be –3.85 or lower.

Key Concepts

1. A derivative is described as linear when the relationship between an underlying factor and the derivative's value is linear in nature (e.g., a forward currency contract). A nonlinear derivative's value is a function of the change in the value of the underlying asset and is dependent on the state of the underlying asset (e.g., a call option).

2. In general, the VaR of a linear derivative is $VaR_p = \Delta VaR_f$, where the derivative's local delta, Δ, is the sensitivity of the derivative's price to a 1% change in the underlying asset's value.

3. The last term of the following Taylor Series approximation adjusts for the curvature of the nonlinear derivative in addition to the slope or delta.
$$f(x) \approx f(x_0) + f'(x_0)(x - x_0) + \frac{1}{2}f''(x_0)(x - x_0)^2$$

4. More complex derivatives such as mortgage backed securities or bonds with embedded options do not have "well-behaved" quadratic functions. The curvature of the function relating the nonlinear derivative's value to the underlying factor changes for different levels of the underlying factor. Thus, the Taylor Series approximation is not sufficient to capture the shift in curvature.

5. The full-revaluation approach calculates the VaR of the derivative by valuing the derivative based on the underlying value of the index after the decline corresponding to an *x*% VaR of the index. This approach is accurate, but can be highly computational.

6. The delta-normal approach calculates the risk using the delta approximation ($VaR_p = \Delta VaR_f$) which is linear or the delta-gamma approximation, $f(x) \approx f(x_0) + f'(x_0)(x - x_0) + \frac{1}{2}f''(x_0)(x - x_0)^2$, which adjusts for the curvature of the underlying relationship.

7. The structured Monte Carlo (SMC) approach simulates thousands of possible movements in the underlying asset and then uses those outcomes to estimate the VaR for a portfolio of derivatives.

8. An advantage of the SMC approach is that it is able to address multiple risk factors by generating correlated scenarios based on a statistical distribution. A disadvantage of the SMC approach is that in some cases it may not produce an accurate forecast of future volatility and increasing the number of simulations will not improve the forecast.

9. Crisis events cause diversification benefits to deteriorate due to a contagion effect that occurs when a rise in volatility and correlation result in a different return generating process for the underlying asset. This creates problems when using simulations for scenario analysis due to the fact that a simulation using the SMC approach cannot predict scenarios during times of crisis if the covariance matrix was estimated during normal times.

10. One approach for stress testing is to examine historical crisis events and assess the impact on the current portfolio. A disadvantage of using historical events for stress testing is that it is limited to only evaluating events that have actually occurred.

11. The historical simulation approach does not limit the analysis to specific events. Under this approach, the entire data sample is used to identify "extreme stress" situations for different asset classes. The advantage of this approach is that it may identify a crisis event that was previously overlooked for a specific asset class. The disadvantage of the historical simulation approach is that it is still limited to actual historical data.

12. An alternative approach is to analyze different predetermined stress scenarios, such as a shift in the yield curve, and then evaluate the effect of the stress scenarios on the current portfolio. An advantage is that the analysis is not limited to the evaluation of risks that have occurred historically. A disadvantage is that the risk measure can be deceptive if correlations among risk factors are not properly modeled.

13. The worst case scenario (WCS) extends VaR risk measurement estimating the extent of the loss given an unfavorable event.

Concept Checkers

1. A call option and a mortgage backed security derivative are good examples of:
   A. a linear and nonlinear derivative, respectively.
   B. a nonlinear and linear derivative, respectively.
   C. linear derivatives.
   D. nonlinear derivatives.

2. Which of the following statements is(are) true?
   I. A linear derivative's delta must be constant for all levels of value for the underlying factor.
   II. A nonlinear derivative's delta must be constant for all levels of value for the underlying factor.

   A. I only.
   B. II only.
   C. Both I and II.
   D. Neither I nor II.

3. Which of the following statements regarding the Taylor Series approximation is(are) true?
   I. The second derivative of the function for the relationship between the derivative and underlying asset estimates the rate of change in the slope.
   II. The Taylor Series approximation can be used to estimate the change in all nonlinear derivative values.

   A. I only.
   B. II only.
   C. Both I and II.
   D. Neither I nor II.

4. Which of the following statements regarding the measurement of risk for non-linear derivatives is(are) true?
   I. A disadvantage of the delta-normal approach is that it is highly computational.
   II. The full-revaluation approach is most appropriate for portfolios containing mortgage backed securities or options with embedded features.

   A. I only.
   B. II only.
   C. Both I and II.
   D. Neither I nor II.

5. Which of the following statements is incorrect? A contagion effect:
   A. occurs with a rise in both volatility and correlation.
   B. causes a different return generating process in the underlying asset.
   C. results from a crisis event.
   D. increases diversification benefits.

Concept Checker Answers

1. **D** A *nonlinear* derivative's value is a function of the change in the value of the underlying asset and is dependent on the state of the underlying asset.

2. **A** The delta of a linear derivative must be constant. The delta, or slope, of a nonlinear derivative changes for different levels of the underlying factor.

3. **A** The Taylor Series of order two is represented mathematically as:

$$f(x) \approx f(x_0) + f'(x_0)(x - x_0) + \frac{1}{2}f''(x_0)(x - x_0)^2$$

The first derivative tells us the delta, or slope of the line. The second derivative tells us the rate of change. The last term including the second derivative captures the convexity or curvature. This approximation is only useful for "well-behaved" quadratic functions of order two.

4. **D** Both the delta-normal and full-revaluation methods measure the risk of nonlinear securities. The *full-revaluation approach* calculates the VaR of the derivative by valuing the derivative based on the underlying value of the index after the decline corresponding to an *x*% VaR of the index. This approach is accurate, but can be highly computational; therefore, it does not work will for portfolios of more complex derivatives such as mortgage-backed securities, swaptions, or options with embedded features. The *delta-normal approach* calculates the risk using the delta approximation, which is linear or the delta-gamma approximation, which adjusts for the curvature of the underlying relationship. This approach simplifies the calculation of more complex securities by approximating the changes.

5. **D** A *contagion effect* occurs with a rise in volatility and correlation causing a different return generating process. Some specific examples of events leading to the breakdown of historical correlation matrices causing a contagion effect are the Asian crisis and the U.S. stock market crash of October 1987. A contagion effect often occurs where volatility and correlations both increase, thus mitigating any diversification benefits.

The following is a review of the Valuation and Risk Models principles designed to address the AIM statements set forth by GARP®. This topic is also covered in:

Operational Risk

Topic 45

Exam Focus

This topic introduces operational risk by defining operational risk and discussing the types of operational risk and bank business lines that must be considered when calculating operational risk capital requirements. Collecting data for loss frequency and loss severity distributions is an important component of allocating operational risk capital among various business lines. Methods for finding the necessary operational loss data points are based on both internal and external data and historical and forward-looking approaches. Concepts such as loss frequency, loss severity, and operational risk insurance are introduced in this topic, but will be examined further in the next topic on extending the value at risk approach to operational risk.

Defining Operational Risk

Some firms define operational risk as all risk that is not credit or market risk. However, most people agree that this definition is far too broad. Past industry definitions of operational risk include:

- Financial risk that is not caused by market risk (i.e., unexpected asset price movements) or credit risk (i.e., the failure of a counterparty to meet financial obligations).
- Any risk developing from a breakdown in normal operations (e.g., system failures or processing mistakes).
- Any risk from internal sources (e.g., internal fraud), excluding the impact of regulatory action or natural disasters.
- Direct or indirect losses that result from ineffective or insufficient systems, personnel, or external events (e.g., natural disasters or political events), excluding business risk (the risk of earnings volatility resulting from business conditions).

In 2001, the Basel Committee on Banking Supervision attempted to incorporate industry views and build a consensus on the definition of operational risk. The Committee's statement of operational risk is as follows:

> *"The risk of direct and indirect loss resulting from inadequate or failed internal processes, people, and systems or from external events."*

The Basel Committee defines operational risk to include internal functions or processes, human factors, systems, firm infrastructure, and outside events. Problems with any of these areas can lead to direct and indirect losses, both expected and unexpected. The operational risk definition explicitly includes **legal risk**, but does not address **reputational risk** or **strategic risk**, presumably because they can be difficult to quantify. This definition concentrates on sources of losses and the impact of operational losses.

Operational Risk Capital Requirements

AIM 45.1: Calculate the regulatory capital using the basic indicator approach and the standardized approach.

The Basel Committee has proposed three approaches for determining the operational risk capital requirement (i.e., the amount of capital needed to protect against the possibility of operational risk losses): (1) the basic indicator approach, (2) the standardized approach, and (3) the advanced measurement approach. The **basic indicator approach** and the **standardized approach** determine capital requirements as a multiple of gross income at either the business line or institution level. The **advanced measurement approach** (AMA) offers institutions the possibility to lower capital requirements in exchange for investing in risk assessment and management technologies.

With the basic indicator approach, operational risk capital is based on 15% of the bank's annual gross income over a 3-year period. Gross income in this case includes both net interest income and noninterest income. For the standardized approach, the bank uses eight business lines with different **beta factors** to calculate the capital charge. With this approach, the beta factor of each business line is multiplied by the annual gross income amount over a 3-year period. The results are then summed to arrive at the total operational risk capital charge under the standardized approach. The beta factors used in this approach are shown as follows:

- Investment banking (corporate finance) 18%
- Investment banking (trading and sales) 18%
- Retail banking 12%
- Commercial banking 15%
- Settlement and payment services 18%
- Agency and custody services 15%
- Asset management 12%
- Retail brokerage 12%

AIM 45.2: Explain the Basel Committee's requirements for the advanced measurement approach (AMA) and their seven categories of operational risk.

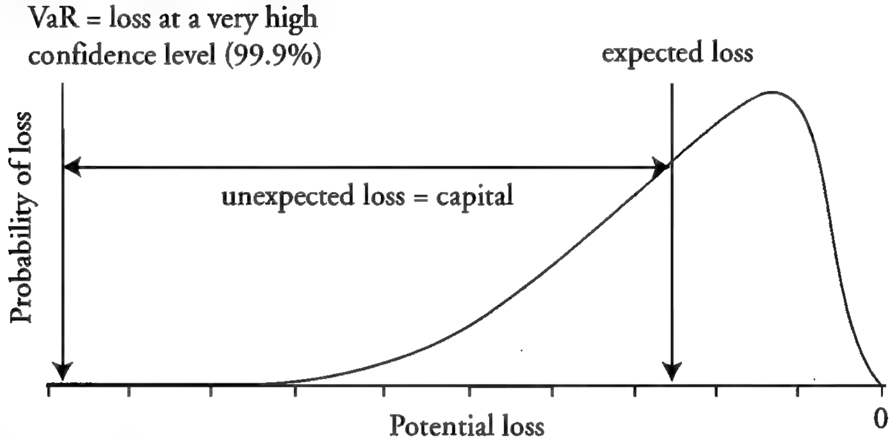
Banks that want to take advantage of the possible lower capital requirements available by using the AMA will be required to determine the operational risk capital charge based on internal criteria that are both qualitative and quantitative in nature. The Basel Committee recommends that large banks move from the standardized approach to the AMA. In order to use either approach, banks must satisfy a number of conditions.

In order to use the standardized approach, banks must: (1) have an operational risk management function that is able to identify, assess, monitor, and control this type of risk, (2) document losses for each business line, (3) report operational risk losses on a regular basis, (4) have a system that has the appropriate level of documentation, and (5) conduct independent audits with both internal and external auditors.

In order to use the advanced measurement approach, banks must satisfy the above requirements in addition to being able to approximate unexpected losses. The calculation of unexpected losses is based on external and internal loss data as well as scenario analysis. With this estimate, the bank is able to find the necessary amount of capital to allocate

to each business line based on the bank's operational value at risk (VaR) measure. The operational risk capital requirement currently proposed by the Basel Committee is equal to the unexpected loss in a total loss distribution that corresponds to a confidence level of 99.9% over a 1-year time horizon. This concept is illustrated in Figure 1.

Figure 1: Capital Requirement



Professor's Note: The calculation of operational risk capital using all three methods will be discussed in more detail in the FRM Part II curriculum.

Operational Risk Categories

The Basel Committee on Banking Supervision disaggregates operational risk into seven types. A majority of the operational risk losses result from clients, products, and business practices.

1. *Clients, products, and business practices.* Failure (either intentional or unintentional) to perform obligations for clients. Examples include mishandling of confidential information, breaches in fiduciary duty, and money laundering.
2. *Internal fraud.* Disobeying the law, regulations, and/or company policy, or misuse of company property. Examples include misreporting data or insider trading. Well-known case studies dealing with internal fraud include Barings, Allied Irish Bank, and Daiwa.
3. *External fraud.* Actions by a third party that disobey the law or misuse property. Examples include robbery or computer hacking.
4. *Damage to physical assets.* Damage occurring from events, such as natural disasters. Examples include a terrorist attack, earthquakes, or fires.
5. *Execution, delivery, and process management.* Failure to correctly process transactions and the inability to uphold relations with counterparties. Examples include data entry errors or unfinished legal documents.
6. *Business disruption and system failures.* Examples include computer failures, both hardware- and software-related, or utility outages.

7. *Employment practices and workplace safety*. Actions that do not follow laws related to employment or health and safety. Examples include worker compensation, discrimination disputes, or disobeying health and safety rules.

Assuming that a bank was active in each of the eight business lines discussed previously, it would have 56 different measures of risk to aggregate into a single operational risk VaR measure. Institutions that are not active in each of the lines of business would require fewer risk measures.

Loss Frequency and Loss Severity

AIM 45.3: Explain how to get a loss distribution from the loss frequency distribution and the loss severity distribution using Monte Carlo simulations.

Operational risk losses can be classified along two dimensions: loss frequency and loss severity. **Loss frequency** is defined as the number of losses over a specific time period (typically one year), and **loss severity** is defined as the value of financial loss suffered (i.e., the size of the loss). It can be reasonably assumed that these two dimensions are independent.

Loss frequency is most often modeled with a **Poisson distribution** (a distribution that models random events). The mean and standard deviation of the Poisson distribution are equal to a single parameter, λ. Over a short time horizon, the probability of losses is then equal to $\lambda \times \Delta t$. Over a time horizon, T, the probability of n losses using this distribution is equal to:

$$e^{-\lambda T} \times \frac{(\lambda T)^n}{n!}$$

The parameter λ is equal to the average number of losses over a given time horizon. So if ten losses occurred over a 5-year period, λ would equal two per year (= 10 / 5).

Loss severity is often modeled with a **lognormal distribution**. This distribution is asymmetrical (the frequency of high-impact, low-frequency losses is not equal to the frequency of low-impact, high-frequency losses) and fat-tailed (rare events occur more often than would be indicated by a normal distribution). The mean and standard deviation are derived from the logarithm of losses.

Loss frequency and loss severity are combined in an effort to simulate an expected loss distribution. The best technique to accomplish this simulation is to use a **Monte Carlo simulation** process. With this process, we make random draws from the loss frequency data and then draw the indicated number of draws from the loss severity data. Each combination of frequency and severity becomes a potential loss event in our loss distribution. This process is continued several thousand times to create the potential loss distribution.

Having created the loss distribution, the desired percentile value can be measured directly. For example, the 99th percentile would correspond with the loss amount that is greater than 99% of the distribution's data. The difference between the losses at the selected percentile

and the mean loss of the distribution equals the unexpected losses at the corresponding confidence level, as was illustrated in Figure 1.

Data Limitations

AIM 45.4: Describe the common data issues that can introduce inaccuracies and biases in the estimation of loss frequency and severity distributions.

The historical record of operational risk loss data is currently inadequate. This creates challenges when trying to accurately estimate frequency and severity. Given the extreme risk that operational problems create, firms are beginning to build a database of potential loss events. Compared to credit risk losses, the data available for operational risk losses is clearly lacking. For example, firms can rely on credit rating agencies to get a clear view of default probabilities and expected losses when assessing credit risk.

It is recommended that banks use internal data when estimating the frequency of losses and utilize both internal and external data when estimating the severity of losses. Regarding external data, there are two data sources available to firms: sharing agreements with other banks and public data.

When incorporating both internal and external operational risk loss data, firms should adjust for inflation. In addition, when viewing external data from other banks it is necessary to use a scale adjustment that applies the loss event to your bank's situation. For instance, if Bank Z has a \$5 million operational risk loss, how would this loss apply to your bank? A simple mathematical proportion will likely over or underestimate the actual loss. As a result, the accepted scale adjustment for firm size is as follows:

$$\text{estimated loss}_{\text{Bank Y}} = \text{external loss}_{\text{Bank Z}} \times \left(\frac{\text{revenue}_{\text{Bank Y}}}{\text{revenue}_{\text{Bank Z}}}\right)^{0.23}$$

Example: Firm size scale adjustment

If the observed loss for Bank Z is \$5 million and it has \$1 billion in revenue, what will be the estimated loss adjusted for firm size for Bank Y, which has revenue of \$2 billion?

Answer:

$$\text{estimated loss}_{\text{Bank Y}} = \$5\text{ million} \times \left(\frac{\$2\text{ billion}}{\$1\text{ billion}}\right)^{0.23} = \$5{,}864{,}175$$

Notice that this loss is much less than the proportional estimate of a \$10 million loss given that Bank Y has twice the revenue.

Scale-adjusted loss data and other data obtained through sharing agreements are useful when constructing a firm's loss severity distribution. Public data, however, is less reliable given the inherent reporting biases. Public loss data likely only contains relatively large

losses from firms that have weak internal controls. As a result, public data is more appropriate when used relative to internal losses. This would involve assigning a multiple to internal data estimates (i.e., mean and standard deviation) that reflects the severity of public external data.

AIM 45.5: Describe how to use scenario analysis in instances when there is scarce data.

Another method for obtaining additional operational risk data points is to use **scenario analysis**. Regulators encourage the use of scenarios since this approach allows management to incorporate events that have not yet occurred. This has a positive effect on the firm since management is actively seeking ways to immunize against potential operational risk losses. The drawback is the amount of time spent by management developing scenarios and contingency plans.

Forward-Looking Approaches

AIM 45.6: Describe how to identify causal relationships and how to use risk and control self assessment (RCSA) and key risk indicators (KRIs) to measure and manage operational risks.

It is important for management to use forward-looking approaches in an attempt to prepare for future operational risk losses. One way to accomplish this objective is to learn from the mistakes of other companies. For example, in Book 1 we learned about a number of financial disasters, including Barings and Allied Irish Bank, which both suffered losses due to rogue traders. Another example is the Hammersmith and Fulham case, where banks took note of a court ruling dealing with counterparty risk.

In the Hammersmith and Fulham case, two traders entered into 600 interest rate swaps totaling 6 billion British pounds over the span of two years. The traders had a low level of understanding of these derivative contracts, and losses quickly grew to millions of pounds. Counterparties became very concerned about the level of credit risk. The auditor at Hammersmith and Fulham was able to void the swap agreements by convincing the court that the traders and, in turn, the company did not have the authority to enter into these transactions. The court agreed with the company and as a result, the swap counterparties were left with unhedged positions and were unable to collect payments from Hammersmith and Fulham.

Causal relationships are a convenient method of identifying potential operational risks. Relationships are analyzed to check for a correlation between firm actions and operational risk losses. For example, if employee turnover or the use of a new computer system demonstrates a strong correlation with losses, the firm should investigate the matter. It is necessary to conduct a cost-benefit analysis if significant relationships are discovered.

One of the most frequently used tools in operational risk identification and measurement is the **risk and control self assessment** (RCSA) program. The basic approach of an RCSA is to survey those managers directly responsible for the operations of the various business lines. It is presumed that they are the closest to the operations and are, therefore, in the best position

to evaluate the risks. The problem with this assumption is that you cannot reasonably expect managers to disclose risks that are out of control. Also, a manager's perception of an appropriate risk-return tradeoff may be different than that of the institution. A sound risk management program requires that risk identification and measurement be independently verified.

The identification of appropriate **key risk indicators** (KRIs) may also be very helpful when attempting to identify operational risks. Examples of KRIs include employee turnover and the number of transactions that ultimately fail. In order to be valuable as risk indicators, the factors must (1) have a predictive relationship to losses and (2) be accessible and measurable in a timely fashion. The idea of utilizing KRIs is to provide the firm with a system that warns of possible losses before they happen.

Scorecard Data

AIM 45.7: Discuss the allocation of operational risk capital and the use of scorecards.

Allocating operational risk capital to each business unit encourages managers to improve their management of operational risks. Less capital will be allocated to those business units that are able to reduce the frequency and severity of risks. The reduction in capital will increase the unit's return on invested capital measure; however, reducing capital may not be ideal if the costs of reducing certain risks outweigh the potential benefits. It is, therefore, necessary for each business line manager to be allocated the optimal amount of operational risk capital.

One method for allocating capital is the **scorecard approach**. This approach involves surveying each manager regarding the key features of each type of risk. Questions are formulated, and answers are assigned scores in an effort to quantify responses. The total score for each business unit represents the total amount of risk. Scores are compared across business units and validated by comparison with historical losses.

Examples of survey questions include: (1) the ratio of supervisors to staff, (2) employee turnover rate, (3) average number of open positions in the business unit at one time, and (4) the presence of confidential information. The objective of the scorecard approach is to make business line managers more aware of operational risks and the potential for losses from those risks. It also encourages senior management to become more involved with the risk management process.

The Power Law

AIM 45.8: Explain how to use the power law to measure operational risk.

The power law is useful in extreme value theory (EVT) when we evaluate the nature of the tails of a given distribution. The use of this law is appropriate since operational risk losses are likely to occur in the tails. The law states that for a range of variables:

$$P(V > X) = K \times X^{-\alpha}$$

where:
V = loss variable
X = large value of V
K and α = constants

The probability that *V* is greater than *X* equals the right side of the equation. The parameters on the right side are found by using operational risk loss data to form a distribution and then using a maximum-likelihood approach to estimate the constants. The power law makes the calculation of VaR at high confidence levels possible since low values of α will represent the extreme tails and, hence, the value at risk from potential operational risks.

Insurance

AIM 45.9: Explain the risks of moral hazard and adverse selection when using insurance to mitigate operational risks.

Managers have the option to insure against the occurrence of operational risks. The important considerations are how much insurance to buy and which operational risks to insure. Insurance companies offer polices on everything from losses related to fire to losses related to a rogue trader. A bank using the AMA for calculating operational risk capital requirements can use insurance to reduce its capital charge. Two issues facing insurance companies and risk managers are moral hazard and adverse selection.

A **moral hazard** occurs when an insurance policy causes an insured company to act differently with the presence of insurance protection. For example, if a firm is insured against a fire, it may be less motivated to take the necessary fire safety precautions. To help protect against the moral hazard issue, insurance companies use deductibles, policy limits, and coinsurance provisions. With coinsurance provisions, the insured firm pays a percentage of the losses in addition to the deductible.

An interesting dilemma exists for rogue trader insurance. A firm with a rogue trader has the potential for profits that are far greater than potential losses, given the protection of insurance. As a result, insurance companies that offer these policies are careful to specify trading limits, and some may even require the insured firm not to reveal the presence of the policy to traders. These insurance companies are also banking on the fact that the discovery of a rogue trader would greatly increase the firm's insurance premiums and greatly harm the firm's reputation.

Adverse selection occurs when an insurance company cannot decipher between good and bad insurance risks. Since the insurance company offers the same polices to all firms, it will attract more bad risks since those firms with poor internal controls are more likely to desire insurance. To combat adverse selection, insurance companies must take an active role in understanding each firm's internal controls. Like auto insurance, premiums can be adjusted to adapt to different situations with varying levels of risk.

Sarbanes-Oxley Act

The corporate accounting scandals in the late 1990s and early 2000s (e.g., Enron bankruptcy) generated increased attention to corporate governance issues. Government officials and investors began losing confidence in corporate reporting practices, which appeared to exacerbate an already shaky market environment for publicly issued securities. The Sarbanes-Oxley Act was enacted in 2002 by the U.S. Congress to prevent the occurrence of questionable reporting and disclosure practices and rebuild the public trust in the financial marketplace. Not only did Sarbanes-Oxley introduce new standards for financial performance accountability, it also mandated substantial penalties for corporate wrongdoing. Sarbanes-Oxley defined a higher level of corporate responsibility, financial performance accountability, and financial reporting transparency.

The Act, in effect, provides greater protection against operational risk. Features of the Sarbanes-Oxley Act include:

Public Company Accounting Oversight Board

- Establishes independent board to oversee public company audits, protect investor interests, and enhance public trust in reporting disclosures.
- Defines the responsibilities of the board.
- Requires public accounting firms to register with the board in order to perform public company audits.

Auditor Independence

- Sets forth actions that strengthen auditor independence.
- Legislates certain activities as unlawful if performed by the external auditor.

Corporate Responsibility

- Requires that audit committees must be independent and undertake specific oversight responsibilities.
- Requires CEO and CFO certification of quarterly and annual financial reports.
- Requires CEO and CFO to forgo bonuses if financial reports need to be restated.
- Provides officer rules of conduct regarding pension blackout periods and other activities.
- Requires the SEC to issue rules for attorneys to report securities laws violations.

Enhanced Financial Disclosures

- Requires a report on the effectiveness of internal controls and procedures for financial reporting.
- Requires code of ethics disclosures for senior financial officers.
- Requires accelerated disclosures involving company security transactions.

Analyst Conflicts of Interest

- Requires the SEC to adopt rules for analyst conflicts when recommendations are made in research reports and public appearances.

Commission Resources and Authority

- Provides additional funding for the SEC.
- Gives the SEC more authority to censure and impose prohibitions on individuals and entities.

Studies and Reports

- Directs federal regulatory bodies to conduct studies regarding consolidation in the accounting industry, credit rating agencies, violations and violators of securities law enforcement actions, and roles of investment banks and investment advisors.

Corporate and Criminal Fraud Accountability

- Provides tougher criminal penalties for securities fraud.
- Makes debts non-dischargeable in the presence of securities fraud.
- Protects employees who provide evidence of fraud (i.e., whistle-blowers).

White Collar Crime Penalty Enhancements

- Persons attempting white collar crime will be prosecuted as if they committed the crime.
- Penalties for mail and wire fraud, as well as Employee Retirement Income Security Act (ERISA) violations, are more severe.
- It requires CEO and CFO certification that periodic financial disclosures comply with Securities Exchange Act of 1934.

Corporate Tax Returns

- CEO should sign the company's federal tax return.

Corporate Fraud and Accountability

- Corporate fraud penalties are harsher.

Key Concepts

1. The Basel definition of operational risk is "the risk of direct and indirect loss resulting from inadequate or failed internal processes, people, and systems or from external events."

2. The three methods for calculating operational risk capital requirements are: (1) the basic indicator approach, (2) the standardized approach, and (3) the advanced measurement approach (AMA). Large banks are encouraged to move from the standardized approach to the AMA in an effort to reduce capital requirements.

3. Operational risk can be divided into seven types: (1) clients, products, and business practices, (2) internal fraud, (3) external fraud, (4) damage to physical assets, (5) execution, delivery, and process management, (6) business disruption and system failures, and (7) employment practices and workplace safety.

4. Operational risk losses can be classified along two dimensions: loss frequency and loss severity. Loss frequency is defined as the number of losses over a specific time period, and loss severity is defined the size of a loss, should a loss occur.

5. Banks should use internal data when estimating the frequency of losses and utilize both internal and external data when estimating the severity of losses. Regarding external data, banks can use sharing agreements with other banks (which includes scale-adjusted data) and public data.

6. Forward-looking approaches are also used to discover potential operational risk loss events. Forward-looking methods include: (1) causal relationships, (2) risk and control self assessment (RCSA), and (3) key risk indicators.

7. Allocating operational risk capital can be accomplished by using the scorecard approach. This approach involves surveying each manager regarding the key features of each type of risk. Answers are assigned scores in an effort to quantify responses.

8. Two issues facing insurance companies that provide insurance for operational risks are moral hazard and adverse selection. A moral hazard occurs when an insurance policy causes a company to act differently with insurance protection. Adverse selection occurs when an insurance company cannot decipher between good and bad insurance risks.

9. Corporate accounting scandals reduce the trust the investing public has in the information released in financial statements. Sarbanes-Oxley is designed to rebuild public trust in financial disclosures and, ultimately, the financial marketplace itself.

Concept Checkers

1. In constructing the operational risk capital requirement for a bank, risks are aggregated for:
   A. commercial and retail banking.
   B. investment banking and asset management.
   C. each of the seven risk types and eight business lines that are relevant.
   D. only those business lines that generate at least 20% of the gross revenue of the bank.

2. According to current Basel Committee proposals, banks using the advanced measurement approach must calculate the operational risk capital charge at a:
   A. 99 percentile confidence level and a 1-year time horizon.
   B. 99 percentile confidence level and a 5-year time horizon.
   C. 99.9 percentile confidence level and a 1-year time horizon.
   D. 99.9 percentile confidence level and a 5-year time horizon.

3. Which of the following is not one of the seven types of operational risk identified by the Basel Committee?
   A. Failed business strategies.
   B. Clients, products, and business practices.
   C. Employment practices and workplace safety.
   D. Execution, delivery, and process management.

4. The Basel definition of operational risk focuses on the risk of losses due to inadequate or failed processes, persons, and systems that cannot protect a company from outside events. The definition has been subject to criticism because it excludes:
   A. market and credit risks.
   B. indirect losses.
   C. failure of information technology operations.
   D. impacts of natural disasters.

5. Which of the following measurement approaches for assessing operational risk would be most appropriate for small banks?
   A. Loss frequency approach.
   B. Basic indicator approach.
   C. Standardized approach.
   D. Advanced measurement approach (AMA).

Concept Checker Answers

1. **C** The construction of the operational risk capital for a bank requires that risks be aggregated over each of the seven types of risk and each of the eight business lines that are relevant for the particular bank.

2. **C** Current Basel Committee proposals require that operational risk capital be calculated at the 99.9th percentile level over a 1-year horizon.

3. **A** Failed business strategies are not included in the definition of operational risk, which includes (1) clients, products, and business practices; (2) internal fraud; (3) external fraud; (4) damage to physical assets; (5) execution, delivery, and process management; (6) business disruption and system failures; and (7) employment practices and workplace safety.

4. **A** The Basel definition excludes credit or market risks. All of the other choices are incorporated in the definition of operational risk.

5. **B** The basic indicator approach is more common for less-sophisticated, typically smaller banks. There is only one indicator of operational risk: gross income.

The following is a review of the Valuation and Risk Models principles designed to address the AIM statements set forth by GARP®. This topic is also covered in:

Extending the VaR Approach to Operational Risk

Topic 46

Exam Focus

This topic introduces various models for measuring and diagnosing operational risk. You should be able to distinguish between high-frequency, low-severity (HFLS) events and low-frequency, high-severity (LFHS) events. In addition to understanding the fundamental elements of the different models, you should know how each model treats HFLS events and LFHS events. The different methods for hedging operational risk, including insurance, self-insurance, and derivatives, such as catastrophe options and catastrophe bonds, are of particular importance.

Measuring Operational Risk

AIM 46.3: Compare and contrast top-down and bottom-up approaches to measuring operational risk.

Operational risk is the potential for legal liabilities and adverse consequences to a firm's reputation or business operations caused by failures of operational processes or the systems that support them.

High-frequency, low-severity (HFLS) **events** are operational losses that occur frequently but that result in small losses.

Low-frequency, high-severity (LFHS) **events** are rare but impose potentially catastrophic costs on the firm. These events can jeopardize the firm's very existence.

A **top-down approach** to operational risk measurement examines the aggregate impact of internal operational failures by estimating the variance of economic variables (such as stock price return, revenue, or costs) that is left unexplained by external macroeconomic factors. As such, it does not distinguish between HFLS and LFHS events.

The top-down approach is relatively simple and is not data-intensive. However, its macro view cannot account for the implementation of new operational risk controls or diagnose specific areas of operational weakness.

By contrast, the **bottom-up approach** analyzes risk in individual processes, which can distinguish between HFLS and LFHS events. Potential effects of new operational risk controls can be modeled and aggregated using correlations between events and processes. As such, bottom-up models can diagnose weaknesses in specific operational procedures and suggest corrections. In this way, they are forward-looking, whereas top-down approaches, relying exclusively on historical data, are backward-looking. Bottom-up approaches are also

more complex and data intensive than top-down approaches. These points are summarized in Figure 1.

Figure 1: Top-Down vs. Bottom-Up Approaches to Operational Risk Measurement

	Top-Down Approaches	*Bottom-Up Approaches*
Sophistication	Simple	Complex
Data requirements	Non-intensive	Intensive
HFLS vs. LFHS	Undifferentiated	Differentiated
Diagnostic ability	No	Yes
Model control solutions	No	Yes
Perspective	Backward-looking	Forward-looking

Top-Down Models

AIM 46.1: Describe the following top-down approaches to measuring operational risks:
- **Multifactor models**
- **Income based models**
- **Expense based models**
- **Operating leverage models**
- **Scenario analysis models**
- **Risk profiling models**

There are six common types of top-down models:

1. Multifactor models.
2. Income-based models.
3. Expense-based models.
4. Operating leverage models.
5. Scenario analysis.
6. Risk profiling models.

Multifactor Models

Using a standard factor model, stock returns can be modeled as:

$$S_{it} = \alpha_i + \beta_{1i}I_{1t} + \beta_{2i}I_{2t} + \beta_{3i}I_{3t} + \ldots + \varepsilon_{it}$$

where:
S_{it} = the return on stock i for period t
I_{jt} = the risk factor index j for period t
β_{ji} = the sensitivity of stock i's stock return to factor j
ε_{it} = the residual estimated through ordinary least squares (OLS) or some other regression technique

In the multifactor model, operational risk is measured as **residual variance**, or $\sigma_\varepsilon^2 = (1 - R^2)\sigma_i^2$, where σ_i^2 is the variance of firm *i*'s equity returns. Because R^2 (the

coefficient of determination) measures the portion of equity return variance explained by the factors I_j, $(1 - R^2)$ measures the unexplained variance that is attributable to operational risk. The presence of LFHS events in the data can distort the results.

Income-Based Models

Income-based models, or earnings risk models, also use a regression model to explain variation in income, such as:

$$INC_{it} = \alpha_i + \beta_{1i}I_{1t} + \beta_{2i}I_{2t} + \beta_{3i}I_{3t} + \ldots + \varepsilon_{it}$$

where:
INC_{it} = the historical reported earnings for firm i for period t

Again, the residual variance is a measure of operational risk to earnings. A similar model could be developed to explain variation in historical revenue, but in either case, these models ignore the opportunity cost of capital and reputation risks that have an impact extending beyond current earnings. Using earnings from different business segments offers some crude diagnostic capability. Again, LFHS events can distort the results.

Expense-Based Models

Rather than relate income to macroeconomic risk factors, expense-based models use normalized historical expense data as the dependent variable in regression models similar to those above. These models are very simple to implement but overlook risks that affect revenues, reputational capital, and the opportunity cost of capital. In addition, they do not properly consider added expenses that control or reduce operational risk. If strategies to reduce operational risk increase expenses, the expense-based model incorrectly considers this an increase in operational risk.

Operating Leverage Models

In this context, **operating leverage** measures the change in variable costs for a given change in total assets. Hence, operating leverage models measure the change in the relationship between variable costs and total assets. **Operating leverage risk** is the risk that this relationship will change such that variable operating expenses will increase by more than asset growth would suggest using historical relationships. Operational leverage models essentially join income and expense models and as a result ignore reputational considerations and the opportunity cost of capital.

Scenario Analysis

The essence of scenario analysis is speculation about the nature and magnitude of catastrophic events (such as regulatory changes, loss of key personnel, massive computer system failure, political coup, or legal action) and estimation of their affect on firm value. It focuses on LFHS events that may not have transpired yet, which represents an advantage over using purely historical data that contain no such events, and it also highlights its subjective nature. It is limited in actually measuring operational risk because assigning probabilities to each event is difficult.

Risk Profiling Models

Risk profiling models relate **operational performance indicators** (such as the number of failed trades, the number of trading errors, the number of incidence reports, and system downtime) to **operational control indicators** (such as training expenditures, outstanding confirmations, percentage of staff vacancies, ratio of supervisors to staff, or average years of staff experience). Analyzing performance indicators measures operational risk by directly examining risky events and, thus, diagnoses operational weaknesses. Analyzing the relationship between performance indicators and control indicators suggests solutions. However, risk indicators may not be appropriate measures of operational risk if there is no direct relationship between operational risk and the indicators.

Bottom-Up Models

AIM 46.2: Describe the following bottom-up approaches to measuring operational risk:
- **Process approaches:**
  - — **Causal networks and scorecards**
  - — **Connectivity models**
  - — **Reliability models**
- **Actuarial approaches:**
  - — **Empirical loss distributions**
  - — **Parametric loss distributions**
  - — **Extreme value theory**

There are three broad categories of bottom-up models:

1. Process approaches.
   - Causal networks.
   - Connectivity models.
   - Reliability models.

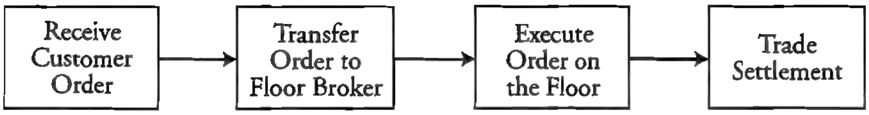
2. Actuarial approaches.
   - Empirical loss distributions.
   - Parametric loss distributions.
   - Extreme value theory (EVT).

3. Proprietary models.

Causal Networks

Causal networks, or *scorecards*, dissect an overall process (like trade execution and settlement in Figure 2) into smaller sequential steps to form what is known as a **process map**. The analyst collects operational performance data for each step to identify high-risk steps on which managers should focus. In Figure 2, for example, the high-risk step for customer-initiated trades is floor execution and might be the focus of risk control efforts. The identification of high-risk steps may depend on the operational performance measure used, which is error rates in this case. Conceivably, other process maps could be created for the individual steps, such as trade settlement.

Figure 2: Process Map for Customer-Initiated Trading



Error Rates

2.5% 1.5% 3.4% 2.3%

Another causal network technique, **event trees**, identifies organizational responses to external events. Different responses occur sequentially in time and depend on the organization's prior response. Different response paths are identified as a process success or failure and focus management's attention on critical risk factors and possible process breakdowns.

Connectivity Models

Connectivity models identify the possible causes of errors in each step of an overall process. For example, errors in the order transfer to the floor broker (step 2 of Figure 2) may be caused by poor line of sight, unreliable electronic communications, or understaffing. By organizing different errors and process failures into an event tree and assigning probabilities to each potential error, the analyst creates a **fault tree analysis**. A major advantage of this approach is that it accommodates interdependencies across steps that make up complex processes.

Reliability Models

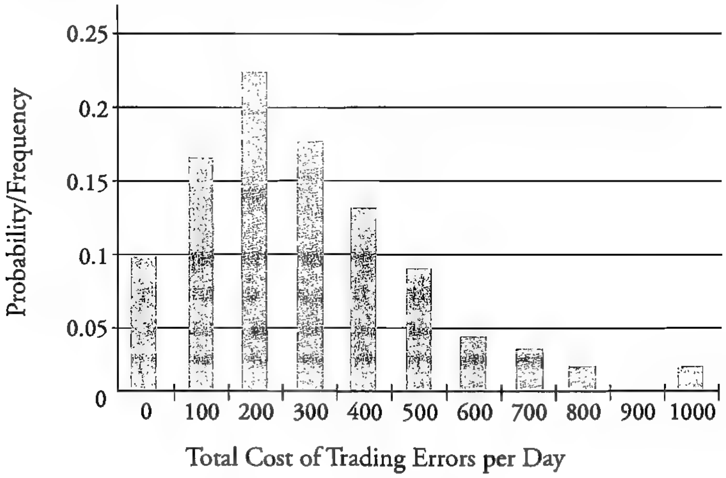
Reliability models estimate the probability of a risk event occurring over a given period of time. Because LFHS events and HFLS events occur with different frequency, by definition, probabilities for each must be estimated separately. Even so, the focus on event frequency ignores the severity of the event.

Empirical Loss Distributions

As a member of the class of actuarial approach models, empirical loss distributions capture both event frequency and event severity, unlike the class of process approaches previously described. They use historical experience to create a simple probability distribution of operational risk costs (i.e., losses), such as the total cost of trading errors per day as shown in Figure 3. The total cost of trading errors, for example, depends on the frequency *and* severity of the errors.

The use of historical experience in empirical loss distributions may overemphasize the firm's specific experience by either overweighting LFHS events that happen to be part of the firm's operational record or by underweighting LFHS events that are not yet reflected in the firm's operating history.

Figure 3: Empirical Loss Distribution



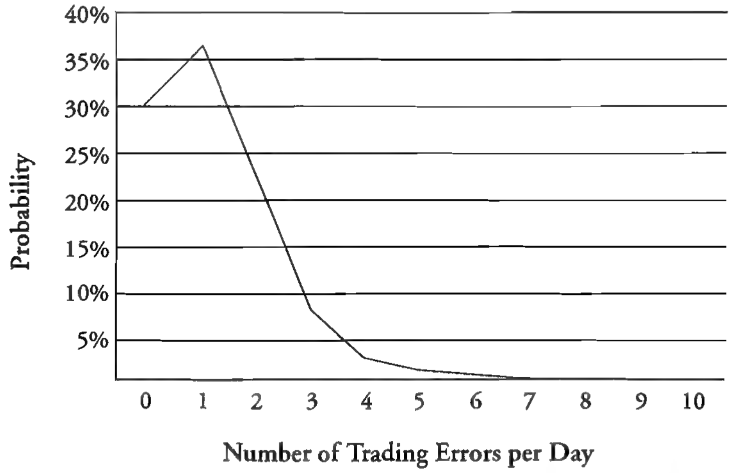
This loss distribution allows a firm to map its expected and unexpected losses due to operational risk. Expected loss due to operational risk typically encompasses the HFLS events (such as the 200 cost in Figure 3). Since this amount of loss is expected to occur, the firm will hold reserves to cover this high-frequency, low-severity loss. However, any losses that have a lower frequency and higher severity are for the most part unexpected losses. Like market risk and credit risk, firms will need to hold enough economic capital to cover unexpected losses up to a high confidence level. Any losses that are greater than the unexpected losses are termed **stress losses** and are generally covered with insurance as will be discussed in the next AIM.

In addition to market risk and credit risk, operational risk can also be evaluated using the VaR metric. Operational VaR (also known as simply OpVaR) is the value that is at risk of loss due to operational factors given a high confidence interval and time horizon. The Basel committee requires that the confidence interval for operational risk be 99.9% and the time horizon be one year. Given the Operational VaR amount and the amount of expected losses, the firm can determine the amount of economic capital needed as a buffer against unexpected losses. The operational risk loss distribution is complicated by the fact that it combines both frequency and severity, both of which are modeled with different probability distributions.

Parametric Loss Distributions

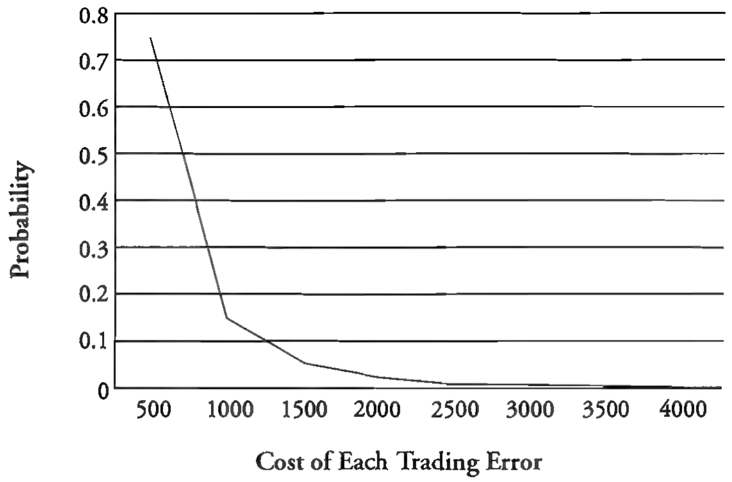
Because total risk is composed of frequency and severity, the analyst must determine a distribution for frequency and severity separately. A **Poisson distribution** is frequently assumed for the distribution of operational risk event frequency. Figure 4 shows a Poisson frequency distribution for the number of trading errors per day having a mean of 1.2 errors per day.

Figure 4: Event Frequency Distribution



The magnitude or severity of each event might also follow a parametric distribution, such as a **lognormal distribution** or a **Weibull distribution**. Figure 5, for example, shows a lognormal distribution of the cost of each trading error. Through a process called **convolution**, these distributions can be combined into a single operational loss distribution similar to Figure 3. Separately identifying frequency and severity distributions permits the analyst to better diagnose risk as stemming from either frequency or severity.

Figure 5: Event Severity Distribution

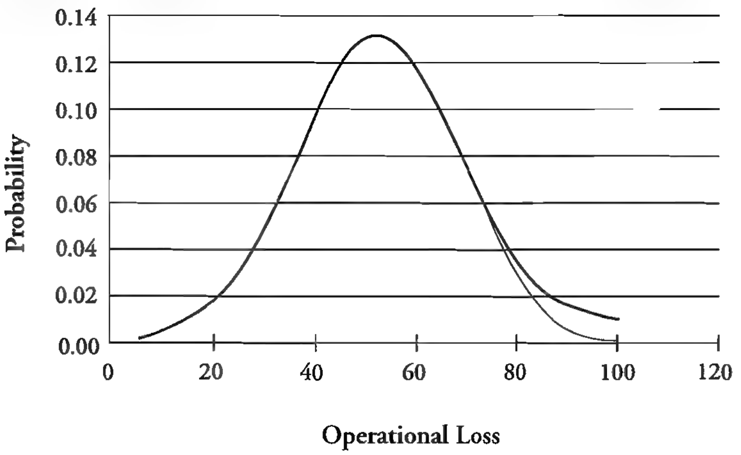


If this analysis is repeated for multiple processes in the firm, which is necessary to estimate total operational risk using a bottom-up approach, the analyst must consider correlations among the different processes. This aggregate analysis requires large amounts of information, a characteristic of bottom-up models in general.

Extreme Value Theory

Often, extreme losses are more likely than standard distributions, such as the lognormal, would suggest, because LFHS events distort empirical distributions. In other words, the right tail in the empirical distribution is generally "fatter" than the right tail in parametric distribution, as shown by the darker line in Figure 6.

Figure 6: Example of a Fat-Tail Distribution of Unexpected Losses



In this case, a different distribution is used to represent the LFHS loss in the right tail. This concept of treating the tails of the distribution in a special manner is known as **extreme value theory** (EVT), and the most common distribution to be used in the tails is the **generalized Pareto distribution** (GPD).

Extreme value theory estimates the expected operational loss of LFHS beyond, say, the 99.9th percentile (i.e., stress losses) using a fat-tailed distribution such as GPD. The expected value of the losses beyond the 99.9th percentile is typically much larger than the 99.9th percentile itself, even assuming a fat-tailed distribution. This approach yields estimates of operational VaR that are much larger than standard value at risk methodologies (such as market VaR) that focus on the percentile itself rather than the expected value of loss beyond a high confidence percentile.

Proprietary Operational Risk Models

Several vendors, such as OpVantage, offer operational risk event databases that can help measure operational risk and cover a period of over ten years. They can fit empirical distributions to parametric distributions, with Monte Carlo simulation to fill in data gaps, and assist in creating causal network models and risk profiles.

Operational Risk Hedging Methods

AIM 46.4: Describe ways to hedge against catastrophic operational losses.

There are three common methods of hedging catastrophic operational losses (i.e., losses greater than expected and unexpected losses given a high confidence level):

1. Insurance.

2. Self-insurance.
    - Cash reserves.
    - Reserves of liquid assets.
    - Contingent credit line.

- Off-shore insurance subsidiary.
- Risk prevention and control.

3. Derivatives.
    - Catastrophe options.
        - Underwriting derivatives.
        - Weather derivatives.
    - Catastrophe bonds.
        - Indemnified notes.
        - Indexed notes.
        - Parametric notes.

Insurance

Firms may wish to transfer operational risk to an insurance company, particularly those firms with relatively little capital to withstand the impact of a LFHS event. Insurance companies write policies to cover many different types of operational risk. For example, Fidelity Insurance protects the firm from employees committing fraudulent acts. Directors' and officers' insurance pays a firm's legal expenses resulting from litigation concerning the fulfillment of officers' and directors' fiduciary duties. Insurance companies diversify these risks by holding large portfolios of policies or by selling a portion of the risk to other insurers in the **reinsurance market**.

However, agents might engage in risky behavior or at least in a less risk-averse manner knowing that an insurance policy will insulate them from the consequences of such behavior. Known as **moral hazard**, this phenomenon increases the cost of insurance and is the reason for deductibles and co-insurance features that cause the insured to share in a portion of the losses, thereby mitigating the moral hazard problem. An insurance policy with deductible and co-insurance features does not cover losses below the deductible amount or above the co-insurance limit.

The breadth of operational risks faced by firms, however, exceeds the risks that insurance policies cover. Because insurance is typically expensive, insurance should be used judiciously to target risks to which the firm is most vulnerable.

Self-Insurance

A potentially less costly alternative is for firms to manage risk internally. Known as **self-insurance**, this method of risk control might entail a firm that is increasing its cash reserves and, hence, equity capital to cover potential operational losses. A variation on this theme would have a firm reserve a portfolio of liquid assets, such as marketable securities, for the same purpose. Another option is for the firm to establish a line of credit that makes financing available contingent on the occurrence of a large operational loss.

Some firms establish wholly owned subsidiaries, or **captive insurers**, outside the United States. This arrangement offers tax advantages because the offshore captive insurer can deduct for tax purposes the discounted value of all future expected losses stemming from a claim spanning several years, whereas the firm can only deduct out-of-pocket losses in the year they are incurred. This tax benefit subsidizes the self-insurance process.

Risk prevention and control is also a form of self-insurance. This can be obtained by using one or more of the operational risk models we previously described.

Hedging Using Derivatives

Derivative securities (such as swaps, forwards, and options) can, in effect, offer insurance; that is, they can provide payments in the event of losses much as an insurance policy would. Unlike the market for derivatives on financial market instruments, the market for operational risk derivatives is not as liquid or well-developed. Nonetheless, securities to hedge operational risk (such as catastrophe options and catastrophe bonds) do exist.

Catastrophe Options and Catastrophe Bonds

AIM 46.5: Describe the characteristics of catastrophe options and catastrophe bonds.

There are two types of catastrophe derivatives useful in hedging operational risk.

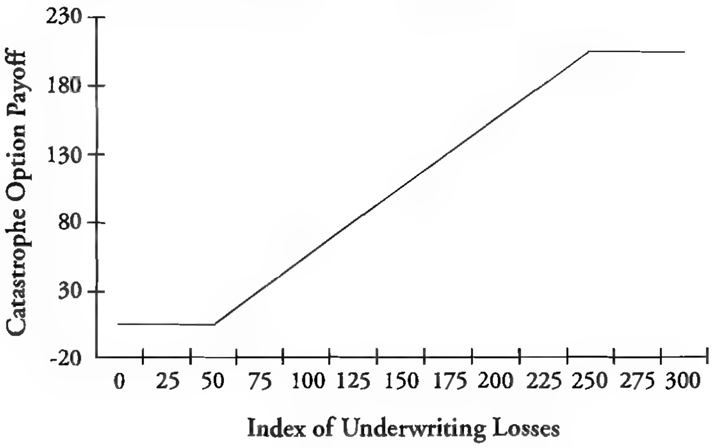
1. Catastrophe options.
   - Underwriting derivatives.
   - Weather derivatives.

2. Catastrophe bonds.
   - Indemnified notes.
   - Indexed notes.
   - Parametric notes.

Catastrophe Options

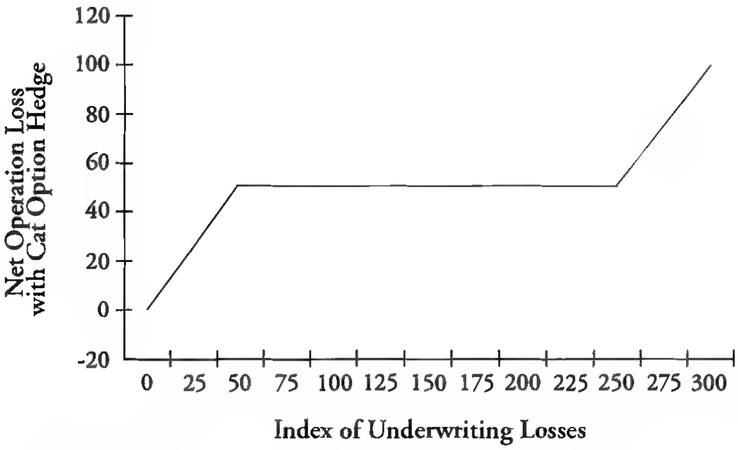
Options provide payoffs to the option holder when the value of an underlying security or index falls above or below a certain value. Theoretically, they can be written on any observable outcome. The Chicago Board of Trade (CBOT) trades catastrophe options, which have payoffs linked to an index of underwriting losses written on a large pool of insurance policies. Unlike a traditional call option, the CBOT catastrophe option is really a spread option in which the payoff has limited rather than unlimited upside potential, as illustrated in Figure 7.

Figure 7: Catastrophe Option



If a firm's operational loss is positively correlated with the index of underwriting losses underlying the "cat" option, the firm's hedged loss is as shown in Figure 8. The catastrophe option does not insulate the firm from all risk. Aside from the basis risk associated with the firm's loss experience being uncorrelated with the underlying index, losses are only insured up to a certain level, at which point the firm absorbs the additional loss. Likewise, small losses are not hedged with cat options either.

Figure 8: Firm Loss With Cat Option Hedge



Note the similarity of the hedged operating losses in Figure 8 to the losses covered by an insurance policy with a deductible and a co-insurance feature. In that case, losses below a deductible amount and above a co-insurance limit are not covered by the policy and are, therefore, absorbed by the insured.

Another category of derivatives potentially related to a firm's operating risk are **weather derivatives**, a steadily growing market. They derive their value from indexes based on weather conditions, such as average temperature, precipitation, and wind speed. These options require valuation models different from those for options on financial securities because the underlying indices exhibit autocorrelation, which traditional models do not accommodate.

Catastrophe Bonds

Bond contracts can be written with embedded options that can be triggered by internal events, external events, or the value of an index. Known as catastrophe bonds, these financing instruments can also hedge a firm's operational risk and come in three types.

1. **Indemnified notes** offer the issuing firm debt relief based on internal events, such as a large underwriting loss for an insurance company. These bonds are subject to the moral hazard problem and require detailed analysis of firm specific risk exposure.

2. **Parametric notes** link cash flows to the magnitude of an external risk event, such as hurricane severity in a particular region. For example, USAA, a leading insurance company, has issued bonds that stop principal and interest payments if a hurricane hits the Gulf of Mexico or the U.S. eastern seaboard.

3. **Indexed notes** provide cash flows related to the value of an independent index, such as a weather index or an insurance underwriting loss index. Because the cash flows to parametric and indexed notes are determined by an external event rather than an internal event, these bonds are not subject to the moral hazard problem.

These so-called "cat" bonds are riskier than straight debt issued by the same firm and are, therefore, almost always noninvestment grade bonds. They also tend to have relatively short maturities (typically less than three years). The low correlation between market risk and catastrophic risk embedded in these securities makes them potentially attractive to investors from a diversification perspective.

Limitations of Operational Risk Hedging

AIM 46.6: Discuss the limitations of hedging operational risk.

Because of its complexity, operational risk management poses some unique challenges. Four limits to operational risk hedging can be identified.

1. **Identifying potential risks.** The fundamental process of identifying the operational risks to be managed is a daunting task.

2. **Subjectivity.** The bottom-up methods often require the analyst to imagine risks that are not yet observable or easily foreseeable. As such, operational risk measurement is a highly subjective process, requiring judgment about how to identify and measure operational risk.

3. **Unanticipated correlations.** Even when various operational risks can be identified, the aggregation of these risks throughout the firm relies on the analyst's assumptions about correlations among these risks. Oftentimes, correlations among operational risk events can deviate from historical experience, particularly in extreme circumstances when problems in one unit can affect other seemingly unrelated units.

4. **Data availability/reliability.** Data to measure and identify operational risk are generally unavailable. Although commercial vendors market databases outlining frequency and severity of operational risk events, these data may not apply to other firms or extrapolate well into the future.

Key Concepts

1. Top-down approaches to operational risk measures differ from bottom-up approaches in that, unlike bottom-up models, top-down models are simpler, require less data, do not distinguish between HFLS and LFHS events, cannot diagnose operational risk or suggest solutions, and are generally backward-looking.

2. There are six different types of top-down models.
   - Multifactor models measure operational risk as the residual variance from a standard multifactor model in which stock returns are a function of various risk factors.
   - Income-based models measure operational risk as the residual variance from a regression model in which income is the dependent variable.
   - Expense-based models measure operational risk as the residual variance from a regression model in which expense is the dependent variable.
   - Operating leverage models measure the change in the relationship between variable costs and total assets.
   - Scenario analysis estimates the effect on firm value caused by hypothetical catastrophic events.
   - Risk profiling models measure the relationship between operational performance indicators and operational control indicators.

3. There are seven different types of bottom-up models.
   - Causal networks deconstruct a process into sequential components with performance data for each step.
   - Connectivity models extend causal networks by identifying possible sources of error for each step.
   - Reliability models estimate the probability of different risk events over a specific period of time.
   - Empirical loss distributions use historical loss data and allow the analyst to examine the distribution of operational losses.
   - Parametric loss distributions fit empirical distributions to standard parametric distributions and often decompose loss distribution into severity distributions and frequency distributions.
   - Extreme value theory uses a special distribution, such as the generalized Pareto distribution, to describe losses in the tail of a fat-tailed distribution.
   - Proprietary models include data and software sold by commercial vendors.

4. Operational risk can be hedged using the following three methods.
   - Insurance allows a firm to transfer its operational risk to a third party for a fee.
   - Self-insurance is a process by which a firm prepares financial reserves to pay for unexpected losses or implements risk prevention and control policies to reduce the probability of unexpected losses.
   - Derivative securities can be used to hedge some operational risks related to underwriting losses, weather, or even internal events.

5. There are two different types of derivative securities for managing operational risk.
   - Catastrophe options are publicly traded securities that have payoffs linked to an index, such as underwriting losses in the insurance industry or weather conditions in a specific geographic area.
   - Catastrophe bonds have cash flows that are linked to an internal operating loss, the magnitude of an external risk event, or the value of an external index.

6. There are at least four limits to operational risk hedging:
   - Its complexity sometimes makes it difficult to identify potential risks, particularly those that have yet to occur.
   - Operational risk analysis is often subjective.
   - When aggregating different risks, it is difficult to estimate the correlations among the individual risks.
   - Operational risk data may be unavailable or unreliable because external data may not be applicable to the firm or because historical data may not extrapolate well into the future.

Concept Checkers

1. Which of the following are characteristics of top-down models of operational risk measurement?
   A. Relatively simple.
   B. Data intensive.
   C. Differentiate between HFLS and LFHS events.
   D. Forward-looking.

2. In bottom-up models of operational risk measurement, operational loss distributions can be:
   A. estimated using historical data only.
   B. estimated entirely using extreme value theory.
   C. estimated using macroeconomic factor models.
   D. constructed from separate frequency distributions and severity distributions.

3. Extreme value theory (EVT):
   A. extends traditional value at risk (VaR) techniques.
   B. is typically used in top-down operational risk models.
   C. analyzes the center and typically tallest part of a loss distribution.
   D. typically yields lower operational VaR estimates than traditional market VaR methodologies.

4. Which of the following statements about insurance and derivatives as a means of hedging operational risk is correct?
   A. Hedging through insurance in inexpensive.
   B. Hedging through derivative securities is subject to the moral hazard problem.
   C. The insurance market is less developed than the market for operational derivative securities.
   D. Insurance policies can be used to hedge a wider array of operational risks than derivative securities.

5. Operational risk hedging is limited because:
   A. no commercial vendors exist to sell operational risk data.
   B. correlations among operational processes are known and fixed.
   C. accurately identifying risks that are not yet apparent is difficult.
   D. the objective nature of the risk assessment creates rigidity in the analysis.

Concept Checker Answers

1. **A** Top-down models rely primarily on aggregate historical data. They are therefore backward-looking and do not differentiate between HFLS and LFHS events because they are pooled together in the data. The aggregated nature of the data also limits the amount of data used in these models.

2. **D** In a process known as convolution, frequency distributions for loss events and severity distributions for loss events can be combined to form an operational loss distribution that captures both HFLS and LFHS events. Factor models can estimate the magnitude of operational risk but do not estimate its distribution. Extreme value theory (EVT) focuses on the tail of a distribution rather than the entire distribution. Historical data can be used to estimate empirical loss distributions; however, they might also be constructed from parametric distributions of frequency and severity distributions fitted to parametric models.

3. **A** Extreme value theory (EVT) focuses on the tail of distributions that do not conform to standard parametric models. These tails in empirical distributions are usually fatter than those of standard parametric distributions and therefore yield large operational risk estimates compared to traditional market VaR methodologies.

4. **D** Although the market for derivative securities on financial instruments is well-developed, the market for operational risk derivatives is much less developed. Consequently, the array of operational risks that can be effectively hedged with operational derivatives is more limited than those offered by insurance. Because the insurance premiums are much greater than claims paid by the insurance industry, insurance is expensive. Its high cost is related, in part, to the moral hazard problem of the insured engaging in risky behavior as a consequence of insurance.

5. **C** Measuring and identifying operational risk is very subjective, in part, because the potential loss may not have yet occurred in any setting and may be difficult for analysts to imagine. This subjectivity is the impetus for various operational risk measurement models that have been developed. Rather than being known and fixed, correlations among various operational processes are often unobservable or difficult to estimate; in addition, they are likely to change in the face of catastrophic events. Vendors selling operational risk data do exist, but the relevance of the data to different firms and its reliability as an indicator of future risks is uncertain.

The following is a review of the Valuation and Risk Models principles designed to address the AIM statements set forth by GARP®. This topic is also covered in:

Stress Testing

Topic 47

Exam Focus

Stress testing focuses on the infrequent but large scale events that occur in the left tail of the return distribution. These are precisely the events that traditional value at risk (VaR) cannot accommodate. The basic idea is to shock key input variables by large amounts (unidimensional analysis) and measure the impact on portfolio value. Multidimensional analysis incorporates the correlation among risk factors and increases the complexity of the analysis significantly. Be able to distinguish between historical and prospective scenarios.

The Role of Stress Testing

AIM 47.1: Describe the purposes of stress testing and the process of implementing a stress testing scenario.

Traditional value at risk (VaR) methodology relies on historical data to generate a distribution of possible future returns. A high confidence level (typically 99%) is chosen to characterize typical market conditions and allows the analyst to make statements such as, "I expect losses to *exceed* the threshold only 1% of the time over the next (arbitrary) time period." Note the emphasis on "exceed" in the previous statement. While VaR is useful for normal market conditions, history clearly tells us that large and unexpected losses do occur (e.g., Black Monday, Russian debt crisis). VaR cannot make predictions about the magnitude of the losses beyond the threshold, and it cannot identify the causes or conditions that can lead to the large losses. The use of **stress testing** addresses these shortcomings in VaR. It is apparent that stress testing should be used as a complement to VaR measures, rather than as a substitute.

Stress testing certainly has an intuitive appeal to the risk manager (identify key input variables that can impact the portfolio) as opposed to probabilistic statements about hypothetical draws from a distribution. In theory, applying stress testing is straightforward: identify the impact variables, assume extreme movements in the identified variables, and compute the new portfolio value. This simplicity is also problematic. While identifying key variables is a reasonable task, trying to predict regime shifts or structural changes is more difficult. Further, these large-scale events are rarely confined to their point of origin and impact other variables complicating the new portfolio valuation. For example, suppose a foreign currency changes from a fixed to floating exchange rate mechanism. Not only will the exchange rate change (appreciate or depreciate relative to other currencies), but interest rates will likely be impacted. These interest rate changes can affect equity markets, and so on. This chain reaction is understandably difficult to foresee and incorporate into a valuation model.

Scenario Analysis

AIM 47.2: Explain the difference in event-driven scenarios and portfolio-driven scenarios.

Generating scenarios can be either event-driven or portfolio-driven. For an **event-driven scenario**, the scenario is generated from events that would likely cause movements in relevant risk factors.

For a **portfolio-driven scenario**, the risk vulnerability of the portfolio is first identified, and is then translated into adverse risk factor movements. For example, an institution that borrows short and lends long would be vulnerable to an increase in interest rates. The scenarios chosen should reflect this vulnerability.

AIM 47.3: Identify common one-variable sensitivity tests.

There are various forms of scenario analysis. With stylized scenarios, the analyst changes one or more risk factors to measure the effect on the portfolio. Rather than having the manager select risk factors, some stylized scenarios are more like industry standards. For example, in *Framework for Voluntary Oversight*[1], the Derivatives Policy Group (DPG) identifies nine specific risk factors to include in stress testing:

1. Parallel yield curve shifts.
2. Changes in steepness of yield curves.
3. Parallel yield curve shifts combined with changes in steepness of yield curves.
4. Changes in yield volatilities.
5. Changes in the value of equity indices.
6. Changes in equity index volatilities.
7. Changes in the value of key currencies (relative to the U.S. dollar).
8. Changes in foreign exchange rate volatilities.
9. Changes in swap spreads in at least the G-7 countries plus Switzerland.

By providing guidelines, these stylized scenarios help managers avoid the "oh no" syndrome, as in, "Oh, no! Why didn't we think of that?" These changes should expose critical portfolio vulnerabilities. Also, by using consistent changes, comparable results will likely be found across institutions.

AIM 47.4: Describe the Standard Portfolio Analysis of Risk (SPAN®) system for measuring portfolio risk.

The **Standard Portfolio Analysis of Risk** (SPAN) system is used to recommend margin requirements on futures and options exchanges. This system focuses on overall portfolio risk rather than risk on a position-by-position basis. The SPAN system estimates the effect on portfolio values of a large set of alternative scenarios. The margin then takes into account the largest loss that the portfolio might suffer.

1 *Framework for Voluntary Oversight*, Derivatives Policy Group, March 1995.

The SPAN system process follows a series of steps. Suppose you had a portfolio of futures and options futures involving the dollar/euro exchange rate. Several scenarios of changes in the dollar/euro price and volatility are identified. Then, the effect (gain or loss) to each future and option on futures in the portfolio is calculated for the scenarios. The risk array is the set of risk array values (i.e., gains or losses for each position) generated from the set of scenarios. The maximum loss for different futures/option on futures positions would occur in differing scenarios. The worst portfolio loss under all scenarios is used to set the margin for the portfolio.

AIM 47.5: Discuss the drawbacks to scenario analysis.

The SPAN system usually employs a small number of risk factors (such as two), which keeps the number of alternative scenarios manageable. With more risk factors, the number of alternative scenarios could easily become unmanageable.

Another drawback of scenario analysis is that it usually assumes that the scenarios are equally probable. This implicitly ignores the correlations between risk factors. Although stress testing does allow risk managers to identify major risks, it is subjective in deciding how serious the risks are. The risk manager could generate an ever larger number of scenarios and uncover more extreme events. But these potential losses might not be significant. Implausible losses might be considered and plausible losses might not be discovered.

AIM 47.6: Explain the difference between unidimensional and multidimensional scenarios.

Scenario analysis involves estimating portfolio value from extreme movements in model inputs. In effect, "shocking" the model parameters in large amounts (i.e., more than would be used in VaR analysis) creates a distribution of portfolio values.

Unidimensional scenario analysis identifies key variables, assumes larger movements than predicted by recent history, and revalues the portfolio. The unstated assumption, now formalized, is to identify the plausible scenarios that would cause the risk factors to change. As mentioned, some typical examples of scenarios involve yield curve shifts, yield curve twists, changes in implied volatility, and changes in currency exchange rates. For example, the Office of Thrift Supervision, which oversees Savings and Loans, requires analysis of market risk in portfolios from parallel shifting of the current yield curve between –400 and +400 basis points.

Typical unidimensional analysis will sequentially test each risk factor searching for potential vulnerabilities. An obvious limitation of this methodology is that the correlation across risk factors is ignored. For example, a drop in interest rates will reduce the funding costs of depository institutions, but will simultaneously increase the prepayments on outstanding mortgages from refinancing. Unidimensional analysis would not consider the additional effects from a decrease in interest rates beyond the impact on funding costs.

Multidimensional scenario analysis attempts to overcome this drawback of unidimensional analysis. The generalized process involves two steps:

Step 1: Hypothesize various states of the world.

Step 2: Consider the impact on all relevant variables.

The multidimensional approach can provide a much richer analysis, but identifying the simultaneous impact on several risk factors is nontrivial. It is not clear how many risk factors are relevant, how correlated the risk factors are, what time period is appropriate to estimate pairwise correlations, and so on. The analyst is further confronted with identifying multiple scenarios and having to assign relative probabilities to them. In addition, the dimensionality of the analysis increases dramatically with additional scenarios, weighting schemes, and risk factor spillover.

AIM 47.7: Compare and contrast various approaches to scenario analysis.

Multidimensional scenario analysis can take two general forms: **historical** or **prospective**. Loosely speaking, the historical approach is backward looking, while the prospective approach is forward looking. At the risk of stating the obvious, **historical scenarios** will examine previous market data to infer the joint movement of key financial variables during times of market stress (e.g., Asian financial crisis in 1997, Black Monday in 1987, terrorist attack on the World Trade Center in 2001). The obvious limitation is the limited number and unique features of each event. A long history is recommended to help provide perspective. For example, on Black Monday, the S&P 500 dropped more than 20% in one day (a statistically large event by any standard), but on a monthly basis, the return was not very different from historical monthly returns.

Prospective scenarios are hypothetical based on reasonable and relevant scenarios that could generate large losses. Examples could include a major earthquake in Tokyo, decrease in oil supply, or increased conflict in the Middle East.

Prospective scenarios are either factor push or conditional. The **factor push method** addresses multidimensionality by "pushing" each risk factor up or down the same amount in the direction that would cause adverse price effects. The magnitude is assumed uniform for all variables (fix the significance level at α). This method is easy to implement by pushing each factor up or down, say 2.33 standard deviations, as appropriate, and measuring the price impact. Additionally, a worst-case scenario can be generated by simultaneously pushing each factor in its adverse direction. The drawback to this method is that the correlation between risk factors is ignored. For example, if two variables are positively correlated, it would not make sense to consider individual movements in opposite directions. Additionally, this naïve approach does not consider that some positions will suffer the greatest losses when the underlying variable(s) do not move (e.g., option positions like long straddles).

The primary advantage to the **conditional scenario method** is the inclusion of correlations across risk factors. By focusing on changes in a subset of variables (holding the other variables constant), incorporation of the variance-covariance matrix is allowed. Hence, the unchanged variables are "zeroed out."

The source of strength of this method is also a significant weakness. The correlation inputs are computed over the entire sample period, which necessarily includes normal and stressed (hectic) market conditions. If the correlations are time dependent, the estimates may deviate significantly during the stressed period. Consequently, the risk manager can choose to employ correlations that are derived from the hectic market period. For example, the

correlation between stock and bond returns is usually positive. However, during market downturns, investors rush to Treasury bonds, and the relationship exhibits a temporary negative correlation between asset prices.

Sensitivity Analysis

AIM 47.8: Define and distinguish between sensitivity analysis and stress testing model parameters.

Sensitivity analysis is a form of stress testing that examines how model output variation relates to model input variation. Stress testing *model parameters* deals with just the variation in the model's inputs.

For sensitivity analysis, a variety of alternative models can be used to price derivatives. Some of these models might be good approximations for small changes in the current environment, but could break down during large movements in key variables. Regarding stress testing model parameters, volatilities and correlations are critical inputs. A breakdown in the historical variance-covariance matrix would cause VaR analysis to become unreasonable. As a result, stress testing should consider the robustness of the risk forecasts to not only changes in model inputs, but also the model itself.

The most important output of stress testing is knowledge of the sensitivity of your portfolio to various risk factors. For example, results could indicate that your portfolio is far less sensitive to currency movements than previously expected. Assuming you performed the analysis correctly, any money spent in hedging against currency movements would be wasted. The analysis could also indicate sensitivity to factors with which you have had little experience. In this case, you need to hedge against or avoid the factor altogether by restructuring your portfolio. Of course, a definite positive by-product of stress testing is focusing the manager's attention on identifying portfolio risk factors.

Improving Stress Tests

AIM 47.9: Explain how the results of a stress test can be used to improve our risk analysis and risk management systems.

The purpose of stress tests is not to consider and evaluate every contingency but rather to identify potential exposures. It is possible that the magnitude of the loss may be unacceptably large, and management discretion must be used. However, there are several tools at management's disposal to alleviate the problem.

- Buy protection through insurance contracts, credit default swaps, and other derivatives. Note that this may just substitute market risk for counterparty risk.
- Modify portfolio to decrease exposure or diversify.
- Alter business strategy; restructure business lines or product mix.
- Develop contingency plan in case of trigger event.
- Secure alternative funding in liquidity stress.

Key Concepts

1. Stress testing focuses on the infrequent but large scale events that occur in the left tail of the return distribution.
2. VaR is based on normal market conditions and cannot accommodate these left-hand events. Therefore, stress testing should complement, not substitute, VaR analysis.
3. Unidimensional scenario analysis identifies key risk factors, shocks the factor by a large amount, and measures the impact on portfolio value.
4. Unidimensional analysis does not consider correlation across multiple risk factors.
5. Multidimensional analysis incorporates correlation across risk factors, but increases the complexity of the analysis.
6. Multidimensional scenario analysis can be historical (backward looking) or prospective (forward looking).
7. Prospective scenario analysis uses either the factor push method or the conditional scenario method.
8. Factor push shifts each variable in the direction that would adversely impact the portfolio.
9. The conditional scenario method incorporates correlations across a subset of key risk factors.
10. The primary disadvantage to the conditional scenario method is that correlations are computed across periods of normal and stressed time periods. Therefore, the estimated correlations may not hold during a hectic time period.
11. Despite best efforts, the magnitude of loss from stress testing may still be unacceptably large.
12. To alleviate the problem of unacceptably large stress losses, managers can alter product mix, purchase insurance, modify exposures, develop contingency plans, or secure liquidity during stress periods.

Concept Checkers

1. An attraction to stress testing is:
   A. ease of identifying key input variables.
   B. straightforward revaluation of portfolio under stressed conditions.
   C. historical data is readily available to estimate frequency and magnitude of losses.
   D. that it ignores the correlation between input variables and effect on portfolio valuation.

2. Which of the following statements about historical and prospective scenario analysis is correct?
   A. Historical scenario analysis only considers recent history.
   B. Prospective scenario analysis provides a long-term perspective.
   C. The time period and frequency of observations is critical to prospective analysis.
   D. The long history of data does not provide adequate observations to evaluate stress situations.

3. The newly hired risk manager at an investment firm tells his supervisor that he would like to incorporate stress testing in the firm's risk management procedures. The new hire makes four statements. Which of the statements is correct?
   A. Stress testing provides a precise maximum loss level.
   B. Using stress testing procedures will be an excellent replacement for the VaR measures the firm is currently using.
   C. The end result of stress testing is the change in value of the portfolio.
   D. One of the advantages to stress testing is the reliance on the chosen scenarios.

4. Stress testing is considered a:
   A. complement to VaR because of its use of a probability estimate of loss.
   B. complement to VaR because of its lack of a probability estimate of loss.
   C. substitute to VaR because of its use of a probability estimate of loss.
   D. substitute to VaR because of its lack of a probability estimate of loss.

5. All of the following are proper management responses to a large computed stress test except:
   A. arranging additional funding during liquidity stress.
   B. modifying current product mix or asset allocation.
   C. selling protection on one of the underlying risk factors.
   D. developing a contingency plan.

Concept Checker Answers

1. **A** An obvious attraction to stress testing is the identification of important input variables. The process of choosing alternative scenarios and regime shifts is significantly more challenging.

2. **D** The drawback to historical scenario analysis is the lack of available events to model portfolio values in stress situations.

3. **C** Stress testing is a complement to VaR, not a replacement. The end result of stress testing is a change in the value of a portfolio under adverse market conditions; however, this change will not be precise because stress testing relies on estimates and hypothetical scenarios. One of the criticisms of stress testing is that results depend on the scenarios chosen.

4. **B** Stress testing lacks the probability estimation of loss that is central to VaR analysis. Rather, stress testing complements VaR by estimating losses for extreme changes in one or more risk factors.

5. **C** Proper response is to purchase protection.

The following is a review of the Valuation and Risk Models principles designed to address the AIM statements set forth by GARP®. This topic is also covered in:

Principles for Sound Stress Testing Practices and Supervision

Topic 48

Exam Focus

The recent financial crisis has revealed numerous weaknesses in stress testing practices as employed by banks, and by doing so, has intensified the need for improvement in stress testing methodologies, scenarios, and the handling of specific risks. Learning from the identified weaknesses of stress testing practices, banks should introduce numerous improvements. Perhaps, most importantly, banks should incorporate into their stress testing models feedback effects arising from initial shocks (such as mortgage delinquencies) that can spill over to other segments and markets, further increasing correlations among various risks and eventually generating severe consequences for banks, markets, and other entities, both nationally and globally.

Stress Testing in Risk Management

AIM 48.1: Describe the rationale for the use of stress testing as a risk management tool.

Stress testing is an important risk management tool that enables a bank to identify the potential sources of risk, evaluate the magnitude of risk, develop tolerance levels for risk, and generate strategies to mitigate risk. For example, stress testing, as a complementary tool to other risk measures, can be valuable for a bank to determine its potential needs for capital to absorb losses in the event of "stress" conditions (i.e., large shocks). Such advance recognition, evaluation, and planning could be crucial for a bank's effective risk management or even survival.

Moreover, Pillar 1 of Basel II related to minimum capital requirements mandates that banks undertake stress testing for assessing capital adequacy if they are using the Internal Models Approach (IMA) to determine market risk or advanced or foundation internal ratings-based (IRB) approaches to determine credit risk. Pillar 2 of Basel II related to the supervisory review process (SRP) mandates that banks undertake general stress tests. Compliance with Basel II requirements would help banks to correctly assess risk and develop plans to reduce their actual losses. Recent financial turmoil has substantially enhanced the significance of comprehensive, flexible, and forward-looking stress testing, as many banks discovered that they were ill-prepared to identify, assess, and mitigate the severe shock they experienced. Their models and strategies, based on past statistical relationships, did not work well under new and rapidly changing conditions.

Professor's Note: The three Pillars of the Basel II Accord, as well as methods for calculating capital requirements for both credit and market risk, will be addressed in Part II of the FRM program.

AIM 48.2: Describe weaknesses identified and recommendations for improvement in:
- **The use of stress testing and integration in risk governance**
- **Stress testing methodologies**
- **Stress testing scenarios**
- **Stress testing handling of specific risks and products**

AIM 48.3: Describe stress testing principles for banks within:
- **Use of stress testing and integration in risk governance**
- **Stress testing methodology and scenario selection**
- **Principles for supervisors**

Stress Testing and Risk Governance

Weaknesses

Lack of involvement of board and senior management. Banks that were able to deal with recent financial crisis relatively well were the ones that attracted active and comprehensive involvement from the board and senior management in the entire stress testing process. The process involved everything from developing and implementing the plan to applying the results as a continually acceptable strategic planning process. On the other hand, banks that did not have the active involvement of senior management and the board were likely hit hard by the recent financial turmoil.

Lack of overall organizational view. Prior to the recent crisis, stress testing in many banks was conducted by separate units concentrating on a risk function or a business line without taking into consideration the overall impact on the bank under stress conditions. Separate testing created organizational barriers making it difficult to integrate insights and perspectives obtained on a bank-wide basis. Moreover, stress tests were conducted based on routine, standard, and mechanical processes and methodologies which lacked the ability to fully encompass changing business environments and insights gained from various areas of the bank.

Lack of fully developed stress testing. Stress testing for market risks has been conducted for years but stress testing for credit risk is quite recent, and stress testing for other risks (e.g., operational) is still in its infancy. Moreover, prior to the crisis, there was no mechanism in place to identify the correlations among various risks. Therefore, stress testing did not adequately identify correlated exposures and risk concentrations across the bank.

Lack of adequate response to crisis. Stress testing methods were not flexible or effective enough to respond swiftly and comprehensively to the changing conditions as the crisis developed. Investment in information technology was not sufficient, which made it difficult to obtain timely information in order to evaluate and generate new and changing test scenarios.

Recommendations

Stress testing, overall governance, and risk management. A stress testing program should constitute a critical component of a bank's governance and risk management planning. To

achieve this goal, the board and senior management should be actively and fully engaged in the stress testing process. The board has the ultimate responsibility for the stress testing program, while senior management should be responsible for managing, implementing, and reviewing the program. Senior management should ensure that the results of stress tests are used in decision making processes, such as setting the risk tolerance and exposure limits, assessment of strategic options, and planning for liquidity and capital adequacy.

Comprehensive stress testing program. A bank should operate a comprehensive stress testing program, focusing on four major areas:

- Risk identification and management.
- Alternative risk perspectives.
- Liquidity and capital management.
- Communication (both internal and external).

To promote effective risk identification and control, stress testing should be used at various levels and activities, such as risk concentrations, business strategies, credit and investment portfolios, and individual and group lending. To achieve a better understanding of risk, stress testing should provide an independent and complementary assessment to other risk measures, such as **economic capital** and **value at risk** (VaR). Stress tests should simulate unprecedented extreme events (shocks). Use of such stress testing would help to discover correlations among various types of risks on a firm-wide basis. Stress testing should be rigorous and forward-looking in order to identify bank specific or market-wide events that may produce a negative impact on a bank's capital and liquidity positions. Stress testing results and risk management strategies should be communicated internally as well as externally. Such disclosure provides an opportunity for market participants to develop a better understanding of a bank's risk exposure and risk management strategies.

Multiple perspectives and techniques. Stress testing programs involve various phases and steps, including identification of risk events, application of modeling techniques, implementation of test procedures, and use of the results for refining the risk mitigation strategies. In this entire process, there should be extensive consultations, collaborations, and interactions among various senior experts within a bank in order to collect multiple perspectives and synergies. Moreover, banks should use multiple techniques, ranging from sensitivity analysis (examining impact of one risk factor on a bank's performance holding all other factors constant), to scenario analysis (examining various scenarios one at a time), to risk simulation (examining the impact of multiple risk factors and interactions simultaneously).

Written policies and documentation. Procedures and written policies should be adequately documented to govern a stress testing program. Documentation is particularly important for firm-wide stress testing. Documentation should include fundamental items such as test types, methodologies, scenarios, underlying assumptions, results, and mitigation strategies.

Sound infrastructure. To implement the stress testing program effectively under times of stress, a bank should have in place a sound, adequate, and flexible infrastructure as well as data collection arrangements. Such infrastructure should enable a bank to aggregate risk exposures quickly, alter methodologies, generate new scenarios, and conduct ad-hoc stress testing under extreme conditions.

Regular assessment. Evaluation of the effectiveness and robustness of stress testing programs should be conducted on a regular basis, on both a quantitative and qualitative basis. Quantitative evaluation should compare the effectiveness of a bank's testing program with other stress testing programs within and outside the bank, and qualitative evaluation should assess those elements of the program that are based on expert opinions and subjective judgments.

Stress Testing Methodologies

Weaknesses

Inadequate infrastructure. Banks did not have adequate infrastructure (and data collection systems) in place that would enable them to quickly identify risk exposures and aggregate them at an institutional level.

Inadequate risk assessment approaches. Stress testing methods were based on an underlying assumption that risk is generated by known and non-stochastic processes, which would mean that future risk events could be forecasted reasonably well. However, recent turmoil clearly revealed the fallacy of such an assumption and demonstrated that risk assessment approaches that resulted from these methods were ineffective. Historical financial correlations broke down once the crisis started and stress testing models based only on historical data failed to predict the possibility of severe shocks, much less how to cope with these shocks.

Inadequate recognition of interactive effects. The recent financial crisis has generated strong examples of feedback, spillover, and system-wide interactional effects. Stress testing models were unable to capture these effects and did not perform well under a rapidly changing risk characteristics environment.

Initial mortgage default shocks caused an adverse impact on the prices of mortgage-backed securities (MBS), in particular, collateralized mortgage obligations (CMOs). Rising uncertainty about the value of underlying investments (i.e., the pool of mortgages) brought the securitization market to an almost complete halt as activity in origination and distribution of CMOs significantly declined, substantially reducing liquidity. Pipeline risk increased as banks were forced to warehouse loans that they intended to securitize. Funding liquidity risk concerns increased, causing the interbank lending activity to decline significantly. In sum, initial difficulties in the mortgage market caused a decline in funding liquidity, which forced market participants to sell the securities at a significant loss.

Inadequate firm-wide perspective. Stress testing prior to the crisis was mostly geared toward individual business lines, products, and risk exposures without having a comprehensive firm-wide perspective. Firm-wide testing can generate synergistic effects, enabling a bank to better identify and manage risk. For example, a comprehensive stress test based on interactions among various experts across the bank would have enabled traders to recognize in a timely manner the increasing risk of mortgage-backed securities as retail lending units were curtailing their exposure to mortgage lending.

Recommendations

Comprehensive stress testing. Stress testing should be comprehensive, covering business areas and risk exposures as individual entities and on a firm-wide level. It should examine the impact of stress events (shocks) on risk factors while taking into consideration feedback and spillover effects due to correlations. All stress testing activities, in the end, should produce a comprehensive and complete firm-wide risk view.

Risk concentrations. A bank may develop risk concentrations along different dimensions, including concentrations in name, industry, region, single or correlated risk factors, off-balance sheet, contractual or non-contractual (reputational) exposures. Stress testing should enable a bank to identify and control risk concentrations. To achieve this goal, stress testing methodology should be comprehensive and firm-wide as well as focused on a wide array of concentrations, including on-balance sheet and off-balance sheet exposures.

Multiple measures. In order to develop an adequate understanding of the impact of stress events on a bank's overall performance, profitability, operations, and viability, numerous measures should be used. For example, a bank should measure the likely impact of stress conditions on asset and portfolio values, accounting and economic profits (losses), funding gaps, and capital requirements.

Stress Testing Scenarios

Weaknesses

Lack of depth and breadth. Prior to the recent financial turmoil, banks used stress testing scenarios that were ineffective in capturing extreme shocks. Banks employed scenarios based on shocks with mild intensity, shorter duration, and smaller spillover or feedback effects among various markets, assets, and positions. Even when banks used somewhat extreme scenarios, results produced a decline in earnings of 25% or less, however, banks can certainly lose (and have indeed lost) more than 25% in severe stress conditions.

Lack of adequate techniques. Banks have employed numerous techniques, including sensitivity analysis to generate testing scenarios. **Sensitivity analysis** does not take into consideration the feedback effect resulting from correlations among various risk factors, positions, and markets, since it focuses only on the impact of a shock on a single factor at a point in time while holding all other factors constant. Other scenario-generating approaches involve multiple factors, but they are historical or hypothetical in nature.

Lack of forward-looking scenarios. Historical and hypothetical scenarios as used by banks turned out to be less effective in the context of the recent crisis because they were based on shocks with smaller intensity, a shorter length of time, and insignificant risk correlations. Banks rarely discussed the possibility of extreme scenarios, and when it was discussed, such a possibility was quickly ruled out.

Recommendations

A variety of events. Stress testing should cover scenarios encompassing a variety of events and varying severity levels, both at micro and macro levels. That is, scenarios should involve testing both at the single entity (a specific product or business line) level and at the entire firm level.

Futuristic outlook. Scenarios should be developed based on potential future events, emerging risks, new products, and asset and liability composition, rather than historical relationships, which may not continue in the future due to changing risk dynamics and market characteristics.

Synergy effect. In order to develop effective future scenarios, opinions and forecasts should be collected and synthesized from experts and senior management across the bank. The discussion processes should be comprehensive, participative, and well-integrated among various units, products, and business lines.

Time horizon. Stress testing should cover various time horizons along with liquidity conditions. As we have witnessed, liquidity can deteriorate quickly and recessions can continue longer than anticipated. A bank should assess its coping strategies for such occurrences during various time horizons. Underlying assumptions and scenarios can change if the length of time of stress testing has increased. Therefore, a bank should recognize that time horizons can play a critical role in scenario development and testing methodology.

Reverse stress testing. Reverse stress testing involves three phases: outcome, events, and hedging. First, reverse stress testing starts from a scenario with a known outcome, such as severe capital inadequacy, panic deposit withdrawals, or insolvency. Second, an assessment is made as to what kind of events, isolated or correlated, firm or market specific, or other events, can lead to the outcome. Lastly, a bank evaluates the effectiveness of its risk management (e.g., hedging) strategies to cope with the events likely to produce the outcome, including an extreme outcome, such as insolvency. The recent crisis has intensified the need for banks to undertake reverse stress testing in order to enhance the overall effectiveness of their risk management plans to cope with events such as extreme events that may have a low probability of occurrence.

Stress Test Handling

Risks Arising From Complex Structured Products

Complex structured products, such as mortgage-backed securities (MBS), asset-backed securities (ABS), and collateralized debt obligations (CDOs), offer cash flows to investors based on an underlying pool of mortgages, loans, account receivables, corporate bonds, or other financial instruments. For example, in the case of MBSs, mortgage loans are packaged into mortgage pools, and tranches are then issued on the underlying pool where each tranche offers a different risk and return profile.

The use of structured securities has significantly increased over the past decade as a risk management (or investment) tool. Given the characteristics of the structured products, it

should be obvious that the nature, extent, and sources of risk for these securities will be different from non-structured investments that do not have a complex structure, such as regular bonds.

Stress testing of these complex products was not properly conducted based on "severe scenarios." More importantly, banks ascertained the risk of these complex structured securities based on the credit rating of apparently similar cash instruments, such as bonds. The fact of the matter is that these products have a different risk profile, and the credit rating for regular bonds should not have been used to determine the riskiness of these more complex instruments.

There are several recommendations for improving the stress testing of risk arising from the use of complex structured products.

- *A stress test should use all the relevant information about an underlying asset pool.* For example, quality of loans, creditworthiness of the borrower, maturity, and interest rates.
- *Impact of market conditions.* For example, investors are subject to prepayment risk if market mortgage rates decline below the rates on existing mortgages.
- *Contractual obligations.* For example, contingent funding agreements in which firms ensure timely payment of interest and principal if certain agreed upon conditions occur.
- *Subordination level of a specific tranche.* For example, a tranche may offer cash flows only after payment has been made to other tranches, meaning that such a tranche exhibits higher risk, particularly under stress conditions.

Basis Risk

Banks are exposed to various risks that can adversely impact their earnings, asset values, and even solvency. For example, an increase in market interest rates can produce a decline in asset values and interest margins, which in turn can pose challenges to a bank's overall capital adequacy. Banks use numerous risk management tools, including futures contracts, to hedge against potential losses arising from directional risks, such as an inverse relationship between a bank's security portfolio (bonds) and an increase in market interest rates. A bank can engage in a short hedge (selling a futures contract) in order to protect the bond portfolio value in the event of an increase in interest rates. Losses in cash instruments (bonds) are offset by gains in futures markets in the event of an increase in interest rates.

However, successful futures hedging is based upon certain underlying assumptions, including that the basis does not change between opening and closing of a futures position. As shown in the futures material in Book 3, **basis** is the difference between prices or interest rates between the cash market and the futures markets. Changes in basis between opening and closing of futures position is called **basis risk**, which can yield an ineffective hedge. Due to basis risk, instead of completely offsetting the cash market losses through the use of futures contract, a bank may experience a partial offsetting, or in worst case scenarios, no offsetting of losses. Basis risk can arise from various sources, including low correlation between the price movement of underlying cash instruments and the futures contracts, cross hedging, and relative illiquidity of futures contracts.

Banks tend to focus on directional risks but ignore the essence of basis risk while conducting stress testing. Therefore, they could not adequately ascertain the effectiveness of their hedging strategies utilizing futures contracts. Banks can improve stress test handling

by taking basis risk into consideration. The disconnect between futures and cash prices may increase illiquidity under market stress conditions, which can significantly reduce hedging effectiveness. Also, under stress conditions, hedging may turn out to be less successful when multiple entities are trying to pursue the same hedging techniques. Banks should include all of these scenarios, under stressed conditions, to evaluate the effectiveness of hedging strategies.

Counterparty Credit Risk

Banks use numerous tools to effectively manage risk exposures, including swaps, forwards, options, and insurance contracts. For example, banks and dealers can purchase default insurance on their structured credit products, such as collateralized debt obligations. This insurance protection generates cost savings for the issuers. The recent financial crisis, however, produced an unprecedented event. Entities expected to provide protection sustained severe losses in the wake of chaotic market conditions and in turn lost credibility. For example, in the case of **monoline insurers** (which provide default protection insurance to issuers of various securities in a specific industry), there had been no default or downgrading prior to 2007, but the recent crisis produced downgrading or even the default of some of these insurers.

Downgrading of some monoline insurers during the recent financial crisis immediately caused downgrading of numerous issuers that had received protection from these insurers, creating a **wrong-way risk**. A wrong-way risk emerges when the probability of default of counterparties increases as a result of general market conditions (general wrong-way risk). Another example of a wrong-way risk is when a company writes an option on its own stock. So, if the probability of default of the writer increases, the risk exposure of the bank increases as well (specific wrong-way risk). Prior to the recent crisis, bank holding companies engaged in diversified business activities did not focus on wrong-way risk in their stress testing practices. That was certainly a weakness in their stress test handling program. Based on the lessons learned from the recent global financial crisis, it is recommended that banks include the potential risks, arising from financial conditions of counterparties, market spillover, and feedback effects under severe stress conditions.

Pipeline Risk

Securitization, creating investment securities from a pool of underlying assets, is used by banks to expand sources of funding, free-up balance sheet space for higher yielding or safer assets, and generate extra revenue. However, securitization is not without risks. During market stress, a bank may not be able to complete the entire process of selling the securities to the public through the issuer-special purpose entities (SPEs). Consequently, market conditions may force the bank to warehouse underlying assets longer than planned and incur financing costs. In addition, a bank can be exposed to many other risks, including liquidity risk due to lack of access to the securitization market. Such risk is called **pipeline risk** (a.k.a. warehouse risk).

Prior to the recent crisis, banks conducted stress tests based on the assumption that pipeline risk would be minimal or non-existent. That is, banks believed that securitization markets would continue to operate smoothly, and if there were any disruption, it would not exist for long. That was certainly not the case with recent subprime securitization. Given the recent

experience, a bank should include pipeline risk into its scenario stress tests. Regardless of the probability of the securitization of assets, banks should include such exposures in stress testing procedures for effective management of pipeline risk.

Contingent Risk

There are several sources of contingent risk, including the potential risk arising from the process of securitization and creation of off-balance sheet vehicles, such as special purpose entities (SPEs). As mentioned, securitization enables a bank to free-up balance sheet space (by selling loans to SPEs). However, banks are obligated to inject credit and liquidity to off-balance sheet entities due to contractual agreements or reputational concerns. Banks provide support to off-balance sheet entities, even when they are not obligated to avoid a materially adverse impact on their reputation. Therefore, banks expose themselves to the potential risk involved in fulfilling their commitments, contractual or otherwise, to off-balance sheet vehicles. Such risk may intensify if banks are facing tough times themselves.

Prior to the recent crisis, banks' stress testing mechanisms did not take into consideration the contingent risks related to off-balance sheet exposures. Adequate recognition of such risks would have helped banks to avoid concentrations in such exposures. Stress testing should include scenarios to assess the bank's exposure to off-balance sheet commitments, both contractual and reputational. In addition, stress testing should focus on the potential impact of contingent risk on other risk exposures, such as liquidity, credit, and market risks.

Funding Liquidity Risk

Stress testing conducted by banks did not capture the nature, size, duration, and intensity of the recent crisis. It did not assess funding liquidity risk adequately and also failed to recognize the interrelationship between funding liquidity risk and market (or trading) liquidity risk.

Future stress tests should focus on the correlation of various factors under stress conditions, which may increase the risk exposure for banks. For example, a decline in an asset value (or category) may dry up its liquidity, liquidity pressures may intensify due to contractual or reputational concerns, and damage to a bank's financial condition may diminish access to funding markets.

Recommendations to Supervisors

Assess stress testing methods. Supervisors should make frequent and comprehensive assessments of a bank's stress testing procedures. This involves evaluating a bank's compliance with sound stress testing practices and understanding how stress testing impacts strategic decision making at various levels of management. It is also recommended that supervisors share their views on the direction of global financial markets and how a bank can develop forward-looking stress tests to combat the impact of future market crises.

Take corrective actions. In the event that stress testing procedures or analysis is deemed inadequate, a supervisor should push for corrective actions. It is important to continually verify the effectiveness of key stress testing assumptions and their relevance going forward.

Should a deficiency arise, a supervisor may propose a revision to bank policies and/or a reduction of global risk exposures.

Challenge firm-wide scenarios. It is necessary for supervisors to question the use of stress tests that produce unrealistic results or are inconsistent with a bank's risk appetite. Supervisors should also encourage banks to evaluate scenarios that could harm its reputation or strategic planning effectiveness. It is also recommended to test scenarios that could impact individual business lines within the bank.

Evaluate capital and liquidity needs. Under the Basel II Accord, banks should conduct an analysis of their stress tests when assessing both capital requirements and liquidity. It is, therefore, recommended that supervisors consider the potential capital needs of a bank under times of stress. For a robust analysis, supervisors should utilize capital ratios in their assessment of capital adequacy and determine the mobility of capital across business lines. The ability to meet capital requirements during stress scenarios is crucial to ensure that a bank will remain solvent if such an event occurs. If capital appears low, the supervisor could recommend that the bank increase capital above the requirements outlined by the Basel Committee. Liquidity buffers should also be evaluated for times of stress. If liquidity appears inadequate, contingencies should be discussed with senior management.

Apply additional stress scenarios. It is prudent for supervisors to conduct additional stress tests using common scenarios within a bank's jurisdiction. These additional scenarios would complement the bank's existing stress scenarios and should be relatively easy to implement. It should be clear to bank management that these suggested stress exercises are not a substitute to the existing stress tests designed by senior management.

Consult additional resources. In order to expand their knowledge of stress testing, supervisors should consult with other experts to identify potential stress vulnerabilities. Discussing the behavior of other banks within the industry could provide a greater understanding of imbalances created by banks and how those imbalances may impact the financial markets. It is also important for supervisors to evaluate their own performance and acquire new skills if necessary, such as knowledge of updated quantitative models.

Key Concepts

1. Stress testing is an important tool that enables a bank to identify, assess, monitor, and manage risk. Recent financial turmoil has substantially increased the need for flexible, comprehensive, and forward-looking stress testing.

2. Major weaknesses and recommendations for stress testing and integration in risk governances are as follows. Weaknesses: lack of involvement of board and senior management, lack of overall organizational view, lack of fully developed stress testing, lack of adequate response to crisis. Recommendations: Stress testing should form an essential ingredient of overall governance of risk management plan, encompass multiple techniques and perspectives, involve a sound infrastructure and regular assessment, produce written policies and recommendations, and generate comprehensive firm and market-wide scenario testing.

3. Stress testing methodologies were based on inadequate infrastructure, inadequate risk assessment approaches, inadequate recognition of correlation, and inadequate firm-wide perspectives. Given these weaknesses, recommendations for improvement include development of a comprehensive stress testing approach, identification and control of risk concentrations, and multiple measurements of stress impact.

4. Stress testing scenarios lacked depth and breadth because they were based on mild shocks, shorter duration, and smaller correlation effects among various markets, portfolios, and positions.

5. Banks evaluated the risk of complex structured products based on the credit rating of similar cash instruments. However, the nature, magnitude, and sources of risk for these products are different from non-structured products. In order to identify, assess, monitor, and control risk exposure of complex structured products, stress testing plans should utilize all the relevant information about the underlying asset pool, market conditions, contractual obligations, and subordination levels.

6. Hedging can become less effective if the basis changes between the opening and closing of a future's position. Basis risk can intensify due to factors such as low correlation between cash and futures instruments, illiquidity of futures contracts, and cross hedging.

7. Wrong-way risk arises from the probability of the default of a counterparty due to adverse general market conditions and resulting spillover and feedback effects.

8. Due to a lack of access to the securitization market under stress conditions, a bank may be forced to park assets on the balance sheet longer than planned. Such risk is called pipeline risk.

9. Banks provide credit and liquidity to off-balance sheet SPEs due to contractual agreements or reputational concerns, giving rise to contingent risk. Such arrangements could increase a bank's risk exposure.

10. Stress testing did not fully recognize funding liquidity risk and its correlation with other risks. Future stress tests should focus more on correlations of various factors and risks, including funding liquidity risk.

Concept Checkers

1. Prior to the recent crisis, stress testing was primarily based on which of the following characteristics?

   I. Historical or hypothetical scenarios.
   II. Significant system-wide correlations.

   A. I only.
   B. II only.
   C. Both I and II.
   D. Neither I nor II.

2. Stress testing for a bank's securitized exposures should primarily consider which of the following features?
   A. Credit ratings of issuers' bonds.
   B. Credit ratings of issuers' bonds and quality of underlying asset pool.
   C. Quality of underlying asset pool, subordination level of tranches, and systematic market conditions.
   D. Credit ratings of issuers' bonds, quality of underlying asset pool, and systematic market conditions.

3. Which of the following statements related to stress test handling is correct?
   A. Hedging, through futures contracts, can result in significant loss if basis changes between the opening and closing of futures position.
   B. Pipeline risk emerges only due to market conditions.
   C. Reputational risk is not as important as contractual risk.
   D. A bank with large exposures to counterparties should not be concerned about counterparties' exposure to market conditions or specific assets.

4. Which of the following statements related to conducting stress tests is incorrect?
   A. Basel II requires banks to undertake stress tests for assessing capital adequacy at least once a month.
   B. Results of stress testing should be used for strategic business planning purposes.
   C. Stress testing can use sensitivity analysis to assess risk.
   D. Stress testing should be used to identify risk concentrations.

5. Which of the following statements is(are) correct?

   I. Stress testing for credit risk has been conducted for years, whereas stress testing for interest rate risks is quite recent.
   II. During the recent crisis, pipeline risk decreased as banks were able to warehouse loans which they intended to securitize.

   A. I only.
   B. II only.
   C. Both I and II.
   D. Neither I nor II.

Concept Checker Answers

1. **A** Recent turmoil revealed numerous weaknesses in banks' stress taking practices, such as lack of proper recognition of extreme shocks and presence of significant system-wide correlations (feedback and spillover effects) between different markets, risks, and portfolio positions. Shorter test durations and historical or hypothetical scenario-based testing were key weaknesses in stress testing practices. Actual events showed longer duration of stress conditions and breakdown of historical statistical relationships.

2. **C** Banks made a major mistake in assessing the riskiness of securitized products by relying on credit ratings of apparently similar issues, like corporate bonds. Securitized products are complex and possess different characteristics and risk exposures. Therefore, external rating for non-structured products should not be applicable to these products.

3. **A** Future contracts are used to offset cash market losses against the gains in the futures market. Hedging will be effective if the basis does not change between the opening and closing of the futures position. Changes in basis can reduce effectiveness of hedging and can produce significant loss.

4. **A** Basel II does not impose monthly requirements for stress testing.

5. **D** Stress testing for credit risk is quite recent, whereas stress testing for market and interest rate risks has been conducted for years. Pipeline risk increased, not decreased, as banks were forced to warehouse loans on their balance sheet due to deteriorating conditions in securitization markets.

The following is a review of the Valuation and Risk Models principles designed to address the AIM statements set forth by GARP®. This topic is also covered in:

THE RATING AGENCIES

Topic 49

EXAM FOCUS

This topic discusses the relationship among ratings, investment market participants, and the regulatory process. The key points are: (1) uses of credit ratings by financial market participants, (2) the ratings performance for rated securities, and (3) examples of ratings-based regulations and relations between rating agencies and regulatory bodies. For the exam, be familiar with the ratings scales utilized by both Moody's and S&P. Also, understand the relationship between regulators and rating agencies.

USES OF CREDIT RATINGS

AIM 49.1: Describe the role of rating agencies in the financial markets.

The role of **credit rating agencies** is to evaluate the creditworthiness of debt securities issued by corporate and other obligors and also to evaluate the creditworthiness of the issuers. An agency's job is to inform investors of the likelihood that an issuer will pay the promised interest and principal payments from a security.

AIM 49.2: Describe some of the market and regulatory forces that have played a role in the growth of the rating agencies.

In the past, banks have been the primary source of debt capital. Today, the capital markets have largely replaced banks—public financial markets are the purchasers of debt securities. Public financial market participants have a wide range of expertise in evaluating credit, but they often lack the credit expertise of banks. In these markets, borrowers, investors, and regulators depend on ratings supplied by rating agencies in the following ways:

- Borrowers need credit ratings to assure access to capital and a reasonable cost of borrowing.
- Investors use credit ratings to estimate potential losses associated with their debt investments and to evaluate potential risk and return.
- Regulatory agencies use credit ratings to establish capital requirements for broker-dealers, banking and thrift institutions, and insurance companies. Margin requirements can also depend on credit ratings.

The first bond rating agency began about a century ago in the United States, and prospered well into the 1930s. (Credit rating agencies began even earlier, in the mid-19th century). Over time, the number of issuer ratings declined until the 1950s as the creditworthiness of issues improved. Since the 1950s, the role of rating agencies has increased dramatically for several reasons. The use of ratings for regulatory purposes became established and widespread. The number of corporate issuers increased. The breadth of instruments and

obligors spread far beyond bonds, to include asset-backed securities, commercial paper, municipal bonds, counterparty risk, insurance companies, and other credit risks.

Professor's Note: Nationally recognized statistical rating organizations (i.e., Moody's, Standard and Poor's, Fitch Ratings, etc.) are often referred to as "NRSROs."

Interpreting Credit Ratings

AIM 49.3: Describe what a rating scale is, what credit outlooks are, and the difference between solicited and unsolicited ratings.

- **Identify Standard and Poor's and Moody's rating scales and distinguish between investment and noninvestment grade ratings.**

A rating scale is a series of categories that can be applied to investments (or companies), ranking them from very creditworthy to in default. The investments in one category, typically, are less creditworthy than the next higher category and, similarly, more creditworthy than the next lower category. The rating scales for Moody's and Standard and Poor's are shown in Figure 1 to follow.

A credit outlook often accompanies a rating, indicating the likely direction of a future credit change. If the outlook is positive, neutral, or negative, the agency is signaling that the next change in rating would be to raise, leave unchanged, or lower the rating. If the outlook is developing/evolving, the rating agency is signaling that a change is likely, but it is unsure of the direction.

While investors (i.e., subscribers) used to pay the rating agencies for their services, issuers now pay for their ratings. Publicly registered securities are nearly always rated. In some cases, a rating agency will assign ratings without a request from the issuer. These ratings are called **unsolicited** (a.k.a. agency-initiated) **ratings**. If a rating is unsolicited, the rating agency will disclose this fact. Whether solicited or unsolicited, the rating agency will offer to meet with the issuer and give them the opportunity to appeal their ratings.

Moody's and Standard and Poor's credit ratings are the most widely known in the financial markets. In general, the ratings are divided into two main segments: investment grade and speculative grade. **Investment grade** security ratings indicate those firms with adequate repayment capacity. These firms are of the highest quality and have strong financial operations. Many institutional investors are restricted to making investments only in securities ranked as investment grade. The debt issues of firms rated in the **speculative grade** category are those instruments issued by firms with higher risks associated with their ability to repay obligations. Many investors are not allowed to purchase securities rated within the speculative grade category. Securities rated as investment grade sometimes receive ratings downgrades into the speculative rating category, which often necessitate a sale of those **"fallen angels."**

Professor's Note: Recall the discussion of "fallen angels" from the Corporate Bonds topic in Book 3.

Moody's and Standard and Poor's use the following systems to rate the credit risk of issuers. The general ratings given by each rating agency are listed from least credit risk to greatest credit risk.

Figure 1: Rating Scales

Investment Grade	*S&P*	*Moody's*
Highest quality	AAA	Aaa
High quality	AA	Aa
Strong capacity for repayment	A	A
Adequate capacity for repayment	BBB	Baa

Speculative Grade	*S&P*	*Moody's*
Likely to meet obligations with uncertainty	BB	Ba
High-risk obligations	B	B
Currently vulnerable to default	CCC	Caa
	CC	Ca
Lowest quality	C	C
In default	D	

Moody's appends numerical modifiers 1, 2, or 3 to each generic rating classification from Aa through Caa. Standard and Poor's has a similar practice of adding a "+" or a "–" to modify each rating from AA through CCC. Any issue rated BB or below from Standard and Poor's or Ba or below from Moody's is considered speculative grade. So, the demarcation between investment grade and speculative grade is between Baa and Ba for Moody's and BBB and BB for Standard and Poor's.

The Ratings Process

AIM 49.4: Describe the difference between an issuer-pay and a subscriber-pay model and what concerns the issuer-pay model engenders.

Rating agencies typically receive payment from issuers for their rating services. As mentioned earlier, in the **subscriber-pay model**, investors subscribed to the ratings agencies and paid for their services. The **issuer-pay** ("pay-for-rating") **model** is sometimes questioned as having the potential to distort the independence of the rating process. Evidence, however, indicates that this is not the case.

There is a symbiotic relationship between the ratings agencies and the firms receiving ratings that hinges on maintaining the highest independence and objectivity. The rating agency essentially acts as an external monitor of company activity and the accuracy of the ratings reflects the analytical capability of the rating company to measure credit risk. By selling its services, the rating agency is selling its reputation to analyze a firm's ability to repay its obligations. The firms receiving ratings also desire the highest reputation associated with the ratings agencies, because uncertainty in the ratings process would increase their cost of debt. Both parties want the utmost independence and highest reputation associated with the rating process, which serve to minimize any potential influence over the payment

mechanism while simultaneously maximizing rating process credibility. Regulators remain willing to use the ratings from NRSROs for a variety of regulatory purposes. The rating firms have a disciplined research process that is akin to disciplined academic research.

Industrial and Sovereign Debt Ratings

AIM 49.5: Describe and contrast the process for rating industrial and sovereign debt and describe how the distributions of these ratings may differ.

The rating process will differ according to the type of instrument being rated. Additionally, there is variation in the process across rating agencies. The rating process for industrial bonds (following the example of S&P) focuses on the following areas[1]:

- Business risk.
- Industry characteristics.
- Competitive positioning.
- Management.
- Financial risk.
- Financial characteristics.
- Financial policies.
- Profitability.
- Capitalization.
- Cash flow protection.
- Financial flexibility.

Although the ratings agencies do not release their specific rating algorithms, both Moody's and Standard and Poor's indicate that they use industry and firm-specific inputs like those above when determining a firm's bond rating. Some of the factors incorporated in the industry component relate to industry characteristics and competitive factors. The firm specific factors relate to the financial condition of the company and are evaluated using ratio analysis. The weights for the various factors vary, with the heaviest weight on industry risk analysis. The firm's ratios are tracked through time and monitored for potential changes in firm default probability.

Internationally, the sovereign rating will be the ceiling for the rating of an issuer within that country. For sovereigns, there are additional factors to consider such as:

- Political stability.
- Social and economic coherence.
- Integration into global economic system.

Within a rating agency, the rating proposed for an issue/issuer is contrasted with other ratings in the same industry and firms in other industries. This is to assure that a rating (such as Baa) has the same meaning across firms and industries.

1 Caouette, John B., Edward I. Altman, and Paul Narayanan. 2008. *Managing Credit Risk*. John Wiley & Sons, Inc.

Corporate Bond Rating Performance

AIM 49.6: Discuss the ratings performance for corporate bonds.

Ratings communicate an opinion about the creditworthiness of an issuer or an issuer's obligation. They should indicate the likelihood and severity of default. Characteristics of ratings performance for corporate bonds are as follows:

- Ratings and corporate default rates are inversely related. This inverse relationship holds for all time periods following the ratings, such as one year, five years, ten years, etc.
- Yield spreads over treasury bonds correlate highly with ratings (i.e., the greater the spread, the lower the credit rating).
- Default rates and ratings remain inversely related throughout the business cycle.
- Default rates for investment grade issues (Baa or better) are substantially lower than default rates for speculative grade issues (Ba or worse).
- Ratings do change, but are usually fairly stable. It is not uncommon for investment grade ratings to have the same rating at the end of the year that they had at the beginning of the year.

Ratings and Regulation

AIM 49.7: Describe the relationship between the rating agencies and regulators and identify key regulations that impact the rating agencies and the use of ratings in the market.

Regulators like the high quality, independence, and widespread use of rating agency opinions. For rating agencies, the acceptance of their opinions by regulators is a strong signal of the quality of their work. Regulators accept the ratings of only a few rating agencies (NRSROs in the United States and the external credit assessment institutions [ECAIs] under Basel II).

In the United States, the Credit Rating Agency Reform Act of 2006 established the process for designating an NRSRO and provided SEC oversight (to assure the continuation of credible ratings). The Committee of European Banking Supervisors (CEBS) provides ECAI recognition, although it does not regulate or license the rating agencies. However, CEBS does apply criteria to the ratings such as objectivity, independence, international access and transparency, disclosure, resources, and credibility.

The relationship between regulators and rating agencies increases the benefits of ratings, which assist markets with reliable, convenient, and low-cost information. This relationship may make it difficult for new rating agencies to compete against the large, established agencies. There are some tensions between regulators and rating agencies about the interpretations of ratings. For example, the default rates of corporate bonds change over the business cycle, while some regulators would like the probabilities of default for a given rating category to be constant over this cycle. For rating agencies to do this, they would have to issue short-term rather than long-term ratings, and they would have to change ratings much more frequently.

AIM 49.8: Discuss some of the trends and issues emerging from the current credit crisis relevant to the rating agencies and the use of ratings in the market.

Regulators and lawmakers are becoming more involved with rating agency activities. Economic problems have become considerably worse as of late. The recent economic turmoil should heighten the scrutiny of rating agencies. In addition, recent events have provided a rich environment for the rating agencies to reexamine and recalibrate their own methods.

Rating agencies are going to remain a major player in capital markets. The current level of uncertainty increases their relevance. Both economic and regulatory forces should accelerate changes in rating agencies—their roles, importance, and sophistication will probably all be enhanced.

Key Concepts

1. Since many financial market participants may not be specialized credit analysts, they rely on credit ratings when making investments. Investors and regulators rely on credit ratings to judge not only risk-return relationships, but also appropriateness and suitability of potential investment instruments.

2. Various ratings scales exist in the marketplace and carry a spectrum of ratings from high to low quality securities. Two popular rating scales have been created by Moody's and Standard and Poor's. In general, all rating scales are broken into two subcategories, investment grade and speculative grade, which indicate ability to repay obligations.

3. Since rating agencies act as external monitors of operating companies, it is in the best interest of all involved for the raters to maintain independence and objectivity when assigning ratings. Although those being rated usually pay a fee for the rating service, the relationship among rating agencies, firms, and investors creates and requires the highest level of independence, which generates and enhances agency reputation.

4. Ratings are generated by analyzing industry and firm-specific characteristics. Competitive conditions, as well as financial conditions of the company, are evaluated through time and compared across firms. Evidence indicates ratings are correlated to default rates and yield spreads, which are very critical to investors.

5. Regulatory entities use ratings to determine eligible investments for regulated institutions. Regulators also use ratings to set capital requirements for financial institutions and to set margin requirements for various derivative securities.

Concept Checkers

1. The demarcation between investment and speculative grade investments made by Moody's can be found between which of the following ratings?
   A. Aaa and Baa.
   B. A and Baa.
   C. Baa and Ba.
   D. Ba and Caa.

2. The independence, objectivity, and reputation of the rating agency are all enhanced by the:
   A. SEC Regulatory Guidelines.
   B. NRSRO status.
   C. synergistic relationship between financial market participants.
   D. ongoing payments made by issuers to ratings agencies.

3. A Moody's rating of A3 is roughly equivalent to a Standard and Poor's rating of:
   A. AA.
   B. A+.
   C. A–.
   D. AAA.

4. Which of the following statements is not consistent with the empirical evidence connecting ratings with issuance characteristics?
   A. Negative correlation between rating and price, all else equal.
   B. Negative correlation between rating and yield, all else equal.
   C. Negative correlation between rating and incidence of default, all else equal.
   D. Clear separation between investment grade and speculative security categories, all else equal.

5. Which of the following statements are not examples of how credit ratings are used by regulators?
   I. The Department of Labor may use credit ratings to determine which securities are eligible for investment by pension funds.
   II. Transactions involving some highly rated securities are exempt from certain securities reporting requirements.

   A. I only.
   B. II only.
   C. Both I and II.
   D. Neither I nor II.

Concept Checker Answers

1. **C** The separation between investment grade and speculative grade securities occurs between Moody's Baa and Ba ratings.

2. **C** All financial market participants, issuers, rating agencies, investors, and regulators benefit from having the highest level of independence and objectivity in the rating process.

3. **C** Both Moody's and Standard and Poor's use modifiers to clarify their generic ratings. A Moody's rating of A3 is equivalent to a Standard and Poor's rating of A–.

4. **A** There is a direct correlation between rating and price: the higher the rating, the higher the price.

5. **D** Both of these items are examples of how regulators use credit ratings. Note that this question is not addressed directly in the notes, but is an example of a "real world application" question.

The following is a review of the Valuation and Risk Models principles designed to address the AIM statements set forth by GARP®. This topic is also covered in:

External and Internal Ratings

Topic 50

Exam Focus

Credit ratings can apply to whole companies or individual investments. Agencies determine external ratings with both qualitative and quantitative methods, and the historical relationship between ratings and subsequent defaults is quite strong. Banks generate their own internal ratings, and they may use an at-the-point approach or a through-the-cycle approach. For the exam, have a general understanding on how external and internal credit ratings are established.

External Credit Ratings

AIM 50.1: Describe external rating scales, the rating process, and the link between ratings and default.

External rating scales are designed to convey information about either a specific instrument, called an **issue-specific credit rating**, or information about the entity that issued the instrument, which is called an **issuer credit rating**, or both. The ratings are typically one-dimensional, and the **ratings scale** goes from the highest rating to the lowest in uniform increments. The highest grade on the scale represents the lowest amount of risk (maximum safety), and each move down the scale represents a reduction in safety, or an increase in risk. An example is Moody's ratings, which starts at Aaa and moves down to Aa, A, Baa, Ba, B, Caa, Ca, and C. Each successive move represents an increase in the expected loss caused by a default on the issue. Although the scale has many levels from Aaa to C, when applied to bonds, there is an important division that occurs between Baa and Ba. Ratings Baa and above are designated **investment grade**, and ratings Ba and below represent **non-investment grade**. Most other external ratings agencies have a similar set boundary between investment and non-investment grade bonds.

Professor's Note: Recall that Standard and Poor's uses a slightly different scale than Moody's. S&P uses the following ratings: AAA, AA, A, BBB, BB, B, CCC, CC, C, and D. The switch from investment grade to non-investment grade (i.e., junk) occurs between BBB and BB. A rating of D is considered a default rating.

The rating process requires the existence of an adequate amount of information and a defined analytical framework that can be applied globally. The **ratings process** usually consists of the following steps:

1. Conducting qualitative analysis (e.g., competition and quality of management).

2. Conducting quantitative analysis, which would include financial ratio analysis.

3. Meeting with the firm's management.

4. Meeting of the committee in the rating agency assigned to rating the firm.

5. Notifying the rated firm of the assigned rating.

6. Opportunity for the firm to appeal or offer new information.

7. Disseminating the rating to the public via the news media.

After the initial rating, the ratings agency monitors the firm and adjusts the rating as needed.

AIM 50.6: Define and explain a ratings transition matrix and its elements.

Researchers have composed tables (known as a transition matrices) that show the frequency of default, as a percent, over given time horizons for bonds that began the time horizon with a given rating. These tables use historical data to report that for bonds that began a 5-year period with an Aa rating, for example, a certain percent defaulted during the five years. These tables demonstrate that the higher the credit rating, the lower the default frequency. An example of a transition matrix can be seen in Figure 1.

Figure 1: Transition Matrix from Moody's

Rating From:	*Rating To:*							
	Aaa	*Aa*	*A*	*Baa*	*Ba*	*B*	*Caa-C*	*Default*
Aaa	91.75%	7.26%	0.79%	0.17%	0.02%	0.00%	0.00%	0.00%
Aa	1.32%	90.71%	6.92%	0.75%	0.19%	0.04%	0.01%	0.06%
A	0.08%	3.02%	90.24%	5.67%	0.76%	0.12%	0.03%	0.08%
Baa	0.05%	0.33%	5.05%	87.50%	5.72%	0.86%	0.18%	0.31%
Ba	0.01%	0.09%	0.59%	6.70%	82.58%	7.83%	0.72%	1.48%
B	0.00%	0.07%	0.20%	0.80%	7.29%	80.62%	6.23%	4.78%
Caa-C	0.00%	0.03%	0.06%	0.23%	1.07%	7.69%	75.24%	15.69%

Example: Ratings Migration

Given the following one-year transition matrix, what is the probability that a B rated firm will default over a two-year period?

Rating From	*Rating To*			
	A	B	C	Default
A	90%	5%	5%	0%
B	5%	85%	5%	5%
C	0%	5%	80%	15%

Answer:

At the end of year 1, there is a 5% chance of default and an 85% chance that the firm will maintain a B rating.

In year 2, there is a 5% chance of default if the firm was rated B after 1 year ($85\% \times 5\% = 4.25\%$). There is a 0% chance of default if the firm was rated A after 1 year ($5\% \times 0\% = 0\%$). Also, there is a 15% chance of default if the firm was rated C after 1 year ($5\% \times 15\% = 0.75\%$).

The probability of default is 5% from year 1 plus 5% chance of default from year 2 (i.e., $4.25\% + 0\% + 0.75\%$) for a total probability of default over a two-year period of 10%.

AIM 50.2: Discuss the impact of time horizon, economic cycle, industry, and geography on external ratings.

Ratings agencies determine the external rating of a firm or bond using current information with the goal of indicating the probability of future events such as default and/or loss. The probability of default given any rating at the beginning of a cycle *increases with the horizon*. The increase in the default rates, or cumulative default rate, is much more dramatic for non-investment grade bonds. In addition to the condition of the firm, forecasted events in the horizon will affect the probabilities. The most notable events are the *economic and industrial cycles*. Since the rating should apply to a long horizon, in many cases, ratings agencies try to give *a rating that incorporates the effect of an average cycle*. This practice leads to the ratings being relatively stable over an economic or industrial cycle. Unfortunately, this averaging practice may lead to an over- or underestimate during periods when the economic conditions deviate too far from an average cycle. Also, the default rate of lower-grade bonds is correlated with the economic cycle, while the default rate of high-grade bonds is fairly stable.

Ratings agencies apply their ratings to different types of firms around the world, and the ratings may be interpreted differently given a specific industry and geographic location. Evidence shows that for a *given rating category, default rates can vary from industry to industry* (e.g., a higher percentage of banks with a given rating will default when compared with firms in other industries with the same rating). However, *geographic location does not seem to cause a similar variation of default* for a given rating class.

One firm may receive different ratings from different agencies. The degree to which ratings discrepancies may exist for a given firm can vary by industry and geographic location. The more ambiguous the data is, the more the ratings tend to vary, and the degree of the data's ambiguity can also vary by industry and geographic location. In addition, the ratings delivered by more specialized and regional agencies tend to be less homogeneous than those delivered by major players like S&P and Moody's.

AIM 50.3: Review the results and explanation of the impact of ratings changes on bond and stock prices.

The evidence supporting the impact of ratings changes on bonds is not surprising:

- *A rating downgrade* is likely to make the *bond price decrease* (stronger evidence).
- *A rating upgrade* is likely to make the *bond price increase* (weaker evidence).

As indicated, the relationship is asymmetric in that the underperformance of recently downgraded bonds is more statistically significant than the over-performance of recently upgraded bonds. Measuring the exact effect can be difficult because a rating change can occur along with other changes in the firm (e.g., a restructuring and changes in the economy such as fluctuating interest rates). Also, some theorize that ratings changes tend to lag the inflow of information, and the market anticipates the change, so the impact on the price is minimized.

For stocks, the change in bond ratings has an even more asymmetric effect on the stock prices than it does on bond prices:

- *A rating downgrade* is likely to lead to a *stock price decrease* (moderate evidence).
- *A rating upgrade* is somewhat likely to lead to a *stock price increase* (evidence is mixed).

The relationship between a change in ratings and the stock price can be complex, and the effect is usually related either to the reason for or the direction of the rating change, or both. A downgrade from a fall in earnings will generally decrease stock prices, but a downgrade from an increase in leverage may leave the price of stocks the same or even increase them. Since firms tend to release good news more readily than bad news, downgrades may be more of a surprise, so downgrades affect stock prices more than upgrades when the firm reveals the good news associated with the upgrade prior to its occurrence.

Evolution of Internal Credit Ratings

AIM 50.4: Compare external and internal ratings approaches.

The core business of a bank is to lend money; however, in order to continually make solid lending decisions, it is increasingly important for banks to create their own internal credit ratings. Today, a bank's process for developing internal credit ratings is largely based on techniques developed and applied by external credit rating agencies. Since external ratings models have been thoroughly tested and validated, it makes sense for banks to apply these techniques when assessing the creditworthiness of their own borrowers.

Previous internal credit ratings approaches were very simplistic, as they often just identified a company as being either a good or a bad borrower. This process lacked the ability to assign unique interest rates based on individual probability of default (PD) and loss given default (LGD). As a result, average interest rates were assigned to good borrowers based on average PDs and average recovery rates.

Two key factors have contributed to the increase in sophistication of internal credit ratings: the growing use of external credit rating agency language in the financial markets and the

encouragement of Basel II rules to refine the approach for calculating credit risk capital requirements. Internal credit ratings models continue to improve, but key issues still exist regarding objectivity, data quality, time horizon, and consistency with external ratings.

Internal Credit Ratings

AIM 50.5: Explain and compare the through-the-cycle and at-the-point internal ratings approaches.

Banks have increasingly been formulating their own internal ratings systems, which can vary from bank to bank. A given bank may have more than one system, such as an **at-the-point approach**, to score a company. This approach's goal is to predict the credit quality over a relatively short horizon of a few months or, more generally, a year. Banks use this approach and employ quantitative models (e.g., logit models) to determine the credit score. Other at-the-point models include those that are based upon arbitrage between debt and equity markets, and structural models.

A bank may also use a **through-the-cycle approach**, which focuses on a longer time horizon and includes the effects of forecasted cycles. The approach uses more qualitative assessments. Given the stability of the ratings over an economic cycle, when using through-the-cycle approaches, high-rated firms may be underrated during growth periods and overrated during the decline of a cycle.

Some evidence suggests that ratings based on at-the-point methodologies tend to vary more over an economic cycle than ratings based upon through-the-cycle methodologies. Generally though, the through-the-cycle and at-the-point approaches are not comparable. The users of the ratings should select the type of rating that suits their goal and horizon. However, some researchers have formulated models that derive longer-term, through-the-cycle ratings from a sufficient history of short-term, at-the-point probability of defaults.

Because of their short-term focus, the use of at-the-point approaches may be *procyclical* (i.e., they tend to amplify the business cycle). The reason for this is as follows:

economic downturn → rating downgrades → decrease in loans and economic activity
economic upturn → rating upgrades → increase in loans and economic activity

Furthermore, the changes in ratings and lending policies can *lag* the economic cycle, so just when the economy hits a trough and is about to start expanding, the banks may downgrade firms and restrict the credit they need to participate in the expansion.

AIM 50.7: Describe the process for and issues with building, calibrating and backtesting an internal rating system.

One method for building an internal rating system is to create ratings that resemble those set by ratings agencies. A bank can accomplish this task by assigning weights to financial ratios and risk factors that have been deemed most important by the rating agency analyst. Internal rating templates can then be constructed to properly score a company (e.g., 0-100). This score is based on the pre-determined weights of important financial ratios and risk factors that contribute to the determination of a company's creditworthiness. To ensure the

weights used are an accurate representation of reality, a comparison of a sample of internal ratings and external ratings is appropriate.

Internal rating systems are established to determine the credit risk of a bank's loans. In addition, they are also used for managing the bank's loan portfolio by assisting with the calculation of economic capital required. In order to accomplish these two objectives, internal ratings systems should properly reflect information from cumulative default probability tables.

However, before banks can link default probabilities to internal ratings, it is necessary to backtest the current internal rating system. Sufficient historical data of 11–18 years is appropriate to properly validate these ratings.[1] If a bank's transition matrix is found to be unstable, then different matrices will need to be constructed. Once a robust rating system is found, the link between the internal ratings and default rates can be established.

AIM 50.8: Identify and describe the biases that may affect a rating system.

An internal rating system may be biased by several factors. The following list identifies the main factors[2]:

- *Time horizon bias*: mixing ratings from different approaches to score a company (i.e., at-the-point and through-the-cycle approaches).
- *Homogeneity bias*: inability to maintain consistent ratings methods.
- *Principal/agent bias*: moral hazard could result if bank employees do not act in the interest of management.
- *Information bias*: ratings assigned based on insufficient information.
- *Criteria bias*: allocation of ratings is based on unstable criteria.
- *Scale bias*: ratings may be unstable over time.
- *Backtesting bias*: incorrectly linking rating system to default rates.
- *Distribution bias*: using an incorrect distribution to model probability of default.

1 Carey and Hrycay. 2001. Parametrizing credit risk models with ratings data, *Journal of Banking and Finance*, 25, 197-270.

2 Servigny and Renault, *Measuring and Managing Credit Risk*. Chapter 2, Appendix 2C. New York: McGraw-Hill, 2004.

Key Concepts

1. The usual steps in the external ratings process include qualitative and quantitative analysis, a meeting with the firm's management, a meeting of the committee in the rating agency assigned to rating the firm, notification of the firm being rated of the assigned rating, an opportunity for the firm to appeal the rating, and an announcement of the rating to the public.

2. Although external ratings have had a fairly good record in indicating relative rates of default, they are designed to be relatively stable over the business cycle (i.e., using an average cycle approach), which can produce errors in severe cycles.

3. Interpreting external ratings may vary based upon the industry but not necessarily on the geographic location of the firm. Ratings delivered by more specialized and regional agencies tend to be less homogeneous than those delivered by major players like S&P and Moody's.

4. Generally for bonds, a ratings downgrade is likely to make the price decrease, and an upgrade is likely to make the price increase. For stocks, a ratings downgrade is likely to lead to a stock price decrease, and an upgrade is somewhat likely to lead to a price increase.

5. The internal at-the-point ratings approach to score a company is usually short-term, uses quantitative models like logit models, and produces scores that tend to vary over the economic cycle.

6. The internal through-the-cycle ratings approach to score a company has a longer horizon, uses more qualitative information, and tends to be more stable through the economic cycle.

7. Internal ratings can have a procyclical effect on the economy since banks often change ratings with a lag with respect to the change in the economy. Thus, after the economic trough has been reached, it is possible that a bank may downgrade a company poised for recovery with the use of additional credit from the bank.

Concept Checkers

1. Which of the following is not part of the external ratings process? A(n):
   A. qualitative assessment.
   B. meeting with the representatives of the firm.
   C. determination of a fair market price of the bond or company.
   D. opportunity for the company being rated to appeal the rating.

2. External credit ratings scales indicate:
   A. the probability of default or the probability of loss.
   B. the probability of default but not the probability of loss.
   C. the probability of loss but not the probability of default.
   D. neither the probability of loss nor the probability default.

3. The longer the time horizon, the higher the incidence of default for a given rating. This effect:
   A. is equal for low-rated and high-rated bonds.
   B. is stronger for low-rated bonds than for high-rated bonds.
   C. is stronger for high-rated bonds than for low-rated bonds.
   D. has not been studied enough to be documented.

4. Given the effort by ratings agencies to incorporate the effect of an average cycle in external ratings, the ratings tend to:
   A. underestimate the probability of default in an economic expansion.
   B. overestimate the probability of default in an economic recession.
   C. underestimate the probability of default in an economic recession.
   D. be unbiased in all phases of the business cycle.

5. With respect to the effect on the price of a bond, the effect of a bond upgrade will:
   A. be positive and stronger than the downward effect of a bond downgrade.
   B. be positive and weaker than the downward effect of a bond downgrade.
   C. have about the same negative effect, in absolute value terms, as a bond downgrade.
   D. be negative and about equal to that of a bond downgrade.

Concept Checker Answers

1. **C** The ratings process does not directly determine prices. The other steps do occur, along with a quantitative assessment, a meeting of the committee in the rating agency assigned to the firm, and the release of the rating to the public.

2. **A** External credit ratings scales indicate either the probability of default, the probability of loss, or both.

3. **B** There is a very strong increase of defaults over time for low-rated bonds. High-rated bonds tend to have much more stable rates of default over time.

4. **C** Because the ratings agencies give ratings that tend to reflect an average business cycle and are generally stable through the cycle, a firm's probability of defaulting during a severe downturn may be underestimated based upon the given rating.

5. **B** A bond's upgrade will have a positive effect on the bond's price, but the negative effect of a bond downgrade is generally stronger.

The following is a review of the Valuation and Risk Models principles designed to address the AIM statements set forth by GARP®. This topic is also covered in:

Country Risk Models

Topic 51

Exam Focus

Country risk can be defined as the delinquency in payment of interest and principal where the delinquency is attributable to the country of the borrower. Country risk modeling is challenging due to the complex relationships among the relevant variables and the lack of reliable and timely data. Assessment tools used to evaluate and compare country risk are discussed in this topic.

Country Risk

AIM 51.1: Define and differentiate between country risk and transfer risk and discuss some of the factors that might lead to each.

AIM 51.2: Define and describe contagion.

Country risk may be defined as the likelihood of delayed, reduced, or omitted payment of interest and principal attributable to conditions of the country of the borrower. It is the broadest measure of credit risk and includes sovereign risk, political risk, and transfer risk. One of the most unfortunate aspects of country risk is that it is contagious—what affects one country tends to affect others. This is referred to as **contagion**.

Transfer risk refers to the economic and financial risk factors associated with a country as distinctly separate from those of the borrower. Transfer risk for a country can be assessed by examining the country's balance of payments. Since foreign exchange is crucial to a country's ability to service its foreign debt, the relationship between a country's foreign exchange movements and its international trade in goods and services (and capital account flows) is critical to the assessment and monitoring of the country's creditworthiness.

Sovereign Risk Analysis

AIM 51.3: Identify and describe some of the major risk factors that are relevant for sovereign risk analysis.

Sovereign risk is the risk that a foreign government, acting in concert with its central bank, will limit or prevent domestic borrowers from repaying debt. In 2006, Standard & Poor's (S&P) developed a framework for evaluating sovereign risk which ranks a country's creditworthiness from one to six in nine distinct categories. These nine categories assess not only willingness to pay, but also ability to pay. The ability to pay is measured by the economic situation of a given sovereign entity, and the willingness to pay is measured by the country's political risk.

Political risk is the relationship between the constitution of a country and the means by which it is enforced. In some countries, the constitution is held in high esteem, while in others, the constitution is used as a vehicle for governments to achieve their goals, not the wants and needs of its citizens. The nature of the bureaucracy in a country contributes to political risk. If a country's bureaucratic process is slow, political risk is relatively high. Certainly, greater potential for corruption in a country's governmental system contributes to the level of political risk for a country.

Economic factors are evaluated to determine a sovereign entity's ability to pay. The categories for measuring economic strength established by S&P are shown below. Note that there are many factors taken into account when establishing a ranking within each category. The following list provides a general idea of what rating agencies are assessing during the ranking process.

- *Income and economic structure*: credit availability; income disparity; stability of financial system.
- *Growth prospects*: rate of growth; degree of savings.
- *Fiscal flexibility*: amount of government revenue and expenditures.
- *Monetary flexibility*: effectiveness of monetary policy actions.
- *Government debt burden*: government debt as a percentage of GDP.
- *External debt burden*: amount of gross external debt.
- *External liquidity*: current account structure; fiscal and monetary influence on external accounts.
- *Contingent liabilities*: health and size of nonfinancial public companies.

Corporate vs. Sovereign Default Rates

AIM 51.4: Compare and contrast corporate and sovereign historical default rate patterns.

Both Moody's and S&P have established analytical frameworks for analyzing sovereign credit worthiness. These rating agencies also both caution that the sample of sovereign entities rated is small (around a 100 for both agencies). Figure 1 compares sovereign and corporate default rates over a 1-year and 5-year period. As you can see, corporate debt and sovereign debt exhibit similar default rate patterns.

Figure 1: Comparing Sovereign and Corporate Default Rates[1]

	One-year Period		*Five-year Period*	
	Sovereign	*Corporate*	*Sovereign*	*Corporate*
AAA	0%	0%	0%	0.3%
BBB	0%	0.2%	5.1%	2.6%
CCC/CC	41.2%	26.3%	58.8%	46.2%

1 Standard & Poor's Risk Solutions CreditPro 7.0. Sovereign ratings from 1975–2006 and corporate ratings from 1981–2006.

Challenges in Assessing Country Risk

AIM 51.6: Describe some of the challenges in country risk analysis.

There are several reasons why country risk assessment is prone to error. First of all, the interrelationship among the relevant variables is so complex that it is a monumental challenge to model outcomes. Another challenge for country risk assessment is data related. Many sovereign and foreign borrowers provide incomplete and/or inaccurate information. So, even with the availability of advanced computer technology and large databases, the garbage-in-garbage-out (GIGO) principal often holds true for country risk models. Furthermore, there is a significant delay in the time it takes information to become available.

Managing Country Risk

AIM 51.5: Describe how country risk ratings are used in lending and investment decisions.

Lenders and investors are increasingly using country risk models to help manage and reduce exposure to country risk. Ratings have allowed for more selective decisions to be made when it comes to choosing among borrowers and/or counterparties. As a result, individual countries are more motivated to improve their ratings in an effort to gain access to more financing. In addition, given country risk ratings, financial institutions are able to better quantify country risk and as a result can improve the effectiveness of capital allocation and hedging strategies as well as take action to mitigate liquidity risk.

Key Concepts

1. Country risk is the likelihood of delayed, reduced, or omitted payment of interest and principal attributable to conditions of the country of the borrower.

2. Transfer risk is the economic and financial risk factors associated with a country.

3. Standard & Poor's developed a framework for evaluating sovereign risk which includes categories such as: political risk, income and economic structure, growth prospects, fiscal and monetary flexibility, government and external debt burden, external liquidity, and contingent liabilities.

4. Political risk is the relationship between the constitution of a country and the means by which it is enforced.

5. Country risk assessment is prone to error because: (1) the interrelationship among the relevant variables is so complex that it is difficult to model, and (2) data is often unavailable and/or inaccurate.

6. Country risk ratings have helped lenders and investors mitigate their exposure to country risk.

Concept Checkers

1. Which of the following is a widely used ratio for measuring government debt burden relative to the size of an economy? The ratio of government debt to:
   A. gross national product.
   B. total foreign exchange reserves.
   C. total exports.
   D. gross domestic product.

2. An element of country risk measurement that is associated with the economic and financial factors attributable to a country as distinctly separate from those of the borrower is:
   A. micro risk.
   B. macro risk.
   C. transfer risk.
   D. translation risk.

3. Which of the following reasons most completely describes why country risk assessment is prone to error?
   A. While data is accurate, it is often incomplete.
   B. Different accounting standards are used in different countries.
   C. Disclosure requirements are inconsistent across international borders.
   D. The exchange rate correlations are unstable during economic downturns.

4. Which of the following statements most accurately describes sovereign risk?
   A. Foreign debtor has insufficient funds to make debt payments.
   B. Foreign firm files bankruptcy and court sets aside the debt obligation.
   C. Foreign creditor accelerates the repayment schedule due to violations of covenants.
   D. Sovereign authority prohibits domestic firms from making payments to foreign creditors.

5. The percentage of defaults among AAA-rated sovereign entities over a 5-year period is roughly:
   A. 0%.
   B. 5%.
   C. 20%.
   D. 40%.

Concept Checker Answers

1. **D** A widely used ratio for measuring government debt burden relative to the size of an economy is the ratio of government debt (gross and net) to GDP.

2. **C** Transfer risk is associated with the economic and financial factors attributable to a country as distinctly separate from those of the borrower.

3. **D** In addition to contagion, there are other reasons why country risk assessment is prone to error. First of all, the interrelationship among the relevant variables is complex and hard to model. Also, many sovereign and foreign borrowers provide incomplete and/or inaccurate information.

4. **D** Sovereign risk addresses the actions of sovereign powers in prohibiting domestic entities from making payments to foreign creditors.

5. **A** The percentage of defaults over a 5-year time span for highly-rated sovereign debt is very low. For AAA-rated debt, the default rate has historically been zero.

The following is a review of the Valuation and Risk Models principles designed to address the AIM statements set forth by GARP®. This topic is also covered in:

Loan Portfolios and Expected Loss

Topic 52

Exam Focus

In this topic, we explore the concept of expected loss and examine the exposure of bank loan portfolios. Expected loss is a function of exposure, loss given default, and the probability of default and unexpected loss is the variation in potential loss around the expected loss level. The bank's exposure is a combination of outstandings (loans, bonds, receivables) and a fraction of committed funds. For the exam, know how to calculate expected loss given adjusted exposure and understand the relationship between outstandings and commitments.

Expected Loss

AIM 52.1: Describe the objectives of measuring credit risk for a bank's loan portfolio.

In their role as financial intermediaries, banks must hold credit-sensitive assets, which by definition have a positive probability of default. However, the distribution of losses is highly skewed. That is, most assets will not default (actual loss = 0), but those that do default will incur large losses (actual loss > expected loss > 0). Therefore, the actual loss will differ significantly from the mathematical expectation of loss. The expected credit loss is defined as:

$$\text{expected loss} = \text{exposure} \times \text{loss given default} \times \text{probability of default}$$

The **expected loss** (EL) represents the decrease in value of an asset (portfolio) with a given exposure subject to a positive probability of default.

Loan vs. Bond Portfolio

AIM 52.3: Distinguish between loan and bond portfolios.

Bonds and bond-like instruments are lending arrangements whereby interest (coupons) is paid periodically with repayment of principal at maturity. Bonds are characterized by less complex lending arrangements than commercial loans. For example, loans may have detailed covenants, indentures, and tax and accounting implications that bonds do not. In addition, a loan portfolio is considerably less liquid than a bond portfolio. This is particularly evident during declining markets where a loan portfolio may have to sell at a considerable discount.

Loan Return

AIM 52.4: Explain how a credit downgrade or loan default affects the return of a loan.

Financial intermediaries are exposed to credit risk by the very nature of lending activities. If the borrower deteriorates in credit quality, the intermediary must bear the increased probability of default and lower expected return. That is, the loan terms are not adjusted for the credit downgrade. In the case of default, the actual return on the loan is drastically reduced. On the other hand, if the borrower's financial condition improves, the bank does not share in the upside as in an equity investment. In fact, it is likely that the borrower will seek to renegotiate or refinance at more favorable terms. Hence, the effect of changing credit quality is asymmetric.

Expected and Unexpected Loss

AIM 52.5: Distinguish between expected and unexpected loss.

Intuitively, expected loss is the average anticipated decline in value over a period of time for a given exposure. However, in any particular period current market conditions can increase or decrease the realized loan loss. Therefore, **unexpected loss** denotes the variability in potential loan loss around the average (expected) loss level. Hence, the unexpected loss has a very natural interpretation as the *standard deviation of expected loss*. While the unexpected loss is a random variable and subject to estimation error, it is more important than expected loss in determining capital adequacy levels. Sound risk management focuses on the unexpected loss to determine the proper buffer level to insulate the bank from unlikely, but entirely possible, large losses.

Commitments, Outstandings, and Covenants

AIM 52.6: Define exposures, adjusted exposures, commitments, covenants, and outstandings:

- **Explain how drawn and undrawn portions of a commitment affect exposure**
- **Explain how covenants impact exposures**

Exposure represents the maximum loss that the bank can suffer from borrower default. In reality, the bank is often able to recover some fraction of the loanable funds, and so the exposure is an overestimate of the loss.

Outstandings (OS) denote the credit extended to the borrower through bonds, loans, or receivables due. In theory, the bank's exposure is total current outstandings. In addition to outstandings, banks often extend additional credit via **commitments** (COM). The commitment represents the total amount the bank is prepared to lend to the borrower (i.e., commitment = outstandings + unused portion of commitment). The bank has an obligation to provide funds at the borrower's discretion up to the full commitment level. Funds exhausted by the borrower are considered drawn and increase the exposure of the bank (i.e., increase outstandings). The undrawn portion does not represent direct exposure,

but rather the borrower's call option to draw on the full credit line in distress. Therefore, the undrawn portion would overestimate the bank's total exposure so a more appropriate exposure estimate is needed.

Covenants are the terms and options of the lending arrangement. Typically, the covenants will specify activities the borrower must do (e.g., maintain financial ratios at prespecified levels) and activities the borrower cannot do (e.g., limit additional borrowings).

If no covenants are in place, a firm in distress has the incentive to draw down on their unrestricted commitment. Therefore, the current outstandings does not adequately measure the bank's likely exposure. Hence, any draw down is essentially a term loan increasing the bank's exposure. If we define α as the fraction of committed funds drawn down given default, then the bank's **adjusted exposure** (AE) is defined as $OS + \alpha \times COM_U$.

Professor's Note: Commitments here is the amount of the unused portion of commitments. Any portion of the commitment that is used becomes part of outstandings. Therefore, the amount of funds the firm will draw down given default is dependent on the unused portion of commitments (i.e., COM – OS).

Under financial distress, it is likely that the firm will draw down completely on its commitments before the cost of funds increases or the bank limits or cuts off the credit line. Cognizant of this possibility, the bank will rationally impose covenants on the commitment, including:

- Lowering the maximum draw down.
- Increasing seniority of outstanding funds.
- Increasing collateral.
- Repricing the loan.

Therefore, careful selection of covenants reduces the adjusted exposure. In addition, the bank must be proactive in detecting and enforcing violations of covenants.

Risky and Risk-Free Parts of Exposure

Suppose a bank asset (e.g., loan) is valued at V_0 at t_0 with maturity date at T_H (horizon). Assuming a simplified two-state default process, there are two possibilities for the value of the bank asset:

(1) no-default and bank asset = V_1

or

(2) default prior to the horizon date

Simply stated, the risky portion of the bank's exposure is $OS + \alpha \times COM_U$ assuming the creditor draws down α of the unused commitment. On the other hand, the risk-free portion of the bank's exposure denotes the $(1-\alpha)$ portion of the unused commitment not drawn upon. In dollar terms, this is simply $(1-\alpha) \times COM_U$.

Figure 1: Risky and Risk-Free Parts of an Exposure

$$V_1 = \begin{cases} OS + \alpha \times COM_U & \text{Risky} \\ (1-\alpha) \times COM_U & \text{Risk-Free} \end{cases}$$

Usage Given Default

AIM 52.7: Define usage given default and how it impacts expected and unexpected loss:

- **Explain credit optionality**

As mentioned previously, the borrower may not fully draw down on the unused commitment even in financial distress. Thus, the risk of total loss on the unused commitment is less than the risk of total loss of the outstandings. It is clear that the estimate of the draw down is necessary to accurately measure the exposure. The percentage of the draw down (α) can also be referred to as the **usage given default** (UGD).

Credit Optionality

The unused commitment can be viewed as a credit option owned by the borrower. In effect, the borrower "bought" the option to draw on the line by paying a "commitment fee," typically a small percentage of the commitment. This fee is non-refundable and grants the borrower the right, but not the obligation, to draw down on the line up to the maximum contracted amount at any time.

Clearly, estimating the expected loss on a portfolio hinges on the estimate of the usage given default. There is no science to determining the exact draw down and the historical evidence is limited. Anecdotally, the draw down (usage given default) increases significantly during distress. To put in perspective, Asarnow and Marker (1995)[1] cite UGD from Citibank by credit rating. For example, UGD for AAA and BBB borrowers is 69% and 65%, respectively. However, the UGD does not appear to follow a simple deterministic function and is a source of concern in parameterizing the expected credit loss as we will see in AIM 52.8.

Computing Expected Loss

AIM 52.2: Define, calculate and interpret the expected loss for an individual credit instrument.

Expected loss on a loan is the same as the equation discussed previously in AIM 52.1, but here we change the notation slightly to reflect that the credit risk stems only from the risky part of the asset value on the horizon date. Accordingly, the expected loss on the horizon date (EL_H) is now defined as:

$$EL_H = AE \times LGD \times EDF$$

1 Asarnow, E., and J. Marker (1995), "Measuring Loss on Defaulted Bank Loans: A 24-Year Study", *Journal of Commercial Lending* 77, pp. 11-23.

Note that this expression is more in line with the revised view of the risky asset. Expected loss is based on adjusted exposure, loss given default, and expected default frequency (EDF) (i.e., probability of default).

Example: Computing expected loss

Suppose XYZ bank has booked a loan with the following characteristics: total commitment of \$2,000,000, of which \$1,200,000 is currently outstanding. The bank has assessed an internal credit rating equivalent to a 1% default probability over the next year. Draw down upon default is assumed to be 75%. The bank has additionally estimated a 40% loss given default. **Calculate** the expected loss.

Answer:

$$EL = AE \times EDF \times LGD$$

$$\begin{aligned}\text{Adjusted exposure (AE)} &= OS + (COM - OS) \times UGD \\ &= \$1{,}200{,}000 + (\$800{,}000 \times 0.75) \\ &= \$1{,}800{,}000\end{aligned}$$

$$\begin{aligned}EL &= \$1{,}800{,}000 \times 0.01 \times 0.40 \\ &= \$7{,}200\end{aligned}$$

Credit Risk Models

AIM 52.8: Describe the process of parameterizing credit risk models and its challenges.

A credit risk model is only as good as its inputs. While we all might agree on the necessary elements of the model, the lack of historical data leaves us with little guidance. Necessary variables for parameterization include:

- Adjusted exposure (outstandings, commitments, usage given default).
- Loss given default (estimate will depend on if the assets are secured or unsecured).
- Expected default frequency.
- Maturity.
- Internal risk class rating.

The challenge lies in determining the exact parameters, since reliable data is scarce, borrower characteristics are hidden and change over time, so the covenants must be studied carefully. For example, outstandings and commitments are observable, but UGD is not. Loss given default is critically dependent on the pledge of collateral. Internal risk ratings may be biased based on the bank's limited observation of its own loan losses rather than the entire universe of loan losses. This problem is further exacerbated for private borrowers, where it is impossible to cross-check with public sources (Moody's, Standard and Poor's). Finally, the empirical studies in this area have not reached a consensus on key variables, including recovery rates.

Factors That Impact Recovery Rates

By definition, LGD = 1 – recovery rate. Therefore, the factors that affect the loss given default will also impact the recovery rates. Not surprisingly, the two key factors are based on the asset's **seniority** and underlying **collateral**. Therefore, careful examination of the covenants is necessary to accurately assess the seniority claim. Seniority status will be impacted by the extent and priority of additional borrowings, restrictions on asset sales, and other potentially adverse actions of the borrower.

The presence of collateral is the second most important factor. Obviously, if the assets are secured, this reduces the loss given default and increases the recovery rate, but the quality of the collateral is the key concern. Estimates of the market value (particularly if it changes over time), liquidity, and uniqueness of the collateral are necessary.

Key Concepts

1. The expected loss (EL) represents the decrease in value of an asset with a given exposure subject to a positive probability of default. The quantity is calculated as EL = exposure × loss given default × probability of default.

2. Bond portfolios are more liquid and have less complicated covenants than corporate loans.

3. Credit downgrade increases the likelihood of default lowering the expected return on the asset. Default represents an actual loss and is significantly greater than the expected (mathematical expectation) loss.

4. Expected loss is the average loss in the asset value over a period of time. The unexpected loss represents the variation in losses around the expected loss (standard deviation).

5. Exposure is the current outstanding credit extended to a borrower.

6. Commitments represent the total credit available to the borrower. The drawn down portion is the outstandings and the unused portion is the remaining available credit.

7. Adjusted exposure is a combination of outstandings and a fraction of the unused commitment: $AE = OS + \alpha \times$ unused commitment.

8. Technically the bank is exposed only to the oustandings but firms in distress often draw down on their unused commitment. Therefore the bank's exposure is better modeled assuming some fraction of the available credit that is drawn upon.

9. Covenants are the terms of the loan designed to protect the bank in case of deterioration of credit quality of the borrower. Covenants therefore reduce the adjusted exposure of the lender.

10. Credit optionality denotes the call option (i.e., right to draw down) the borrower has purchased on the commitment for a "commitment fee," usually a small percentage of the total line.

11. Parameterization of credit models is extremely challenging because of the lack of historical data, difficulty in assessing collateral value, and estimating credit risk levels.

12. Seniority and collateral are the most important factors in assessing recovery rates. More senior claims and higher quality collateral increase recovery rates in default.

Concept Checkers

1. Adjusted exposure is not related to which of the following?
   A. Outstandings.
   B. Recovery rates.
   C. Commitments.
   D. Draw down.

2. Which of the following statements is correct? In default:
   A. actual return < expected return and actual loss < expected loss.
   B. actual return > expected return and actual loss < expected loss.
   C. actual return < expected return and actual loss > expected loss.
   D. actual return > expected return and actual loss > expected loss.

3. Covenants are primarily designed to limit:
   A. draw down on outstandings.
   B. draw down on commitments.
   C. the influence of the bank in default claims.
   D. the borrower's right to renegotiate the terms of the loan.

Use the following information to answer Questions 4 and 5.

Big City Bank has contractually agreed to a $20,000,000 credit facility with Upstart Corp. (this value is the amount of commitment). Upstart will immediately access 60% of the commitment (i.e., 60% of the commitment is currently outstanding). Upstart has very little collateral so Big City Bank estimates a one-year probability of default of 2%. The collateral is unique to its industry with limited resale opportunities so Big City assigns only a 20% recovery rate. Big City has no experience with Upstart and conservatively estimates draw down in default to be 75%.

4. The adjusted exposure for Big City Bank is closest to:
   A. $8,000,000.
   B. $12,000,000.
   C. $15,000,000.
   D. $18,000,000.

5. The expected loss for Big City Bank is closest to:
   A. $68,000.
   B. $72,000.
   C. $272,000.
   D. $288,000.

Concept Checker Answers

1. **B** Recovery rates are directly related to expected losses. Adjusted exposure is the sum of outstandings and a fraction of unused commitments.

2. **C** In default, the actual return on the loan will be less than the ex-ante expected return. In addition, the dollar loss on the loan will exceed the expected loss. In effect, both statements convey the same sentiment.

3. **B** In distress, borrowers have the incentive to draw down on commitments. Covenants are defined to limit the extent of the draw down to limit the bank's exposure.

4. **D** Adjusted exposure $= \text{OS} + (\text{COM} - \text{OS}) \times \text{UGD}$
$= \$12{,}000{,}000 + (\$8{,}000{,}000 \times 0.75)$
$= \$18{,}000{,}000$

5. **D** $\text{EL} = \text{AE} \times \text{EDF} \times \text{LGD}$
$= \$18{,}000{,}000 \times 0.02 \times 0.80$ (Note: recovery rate of 20% implies LGD of 80%)
$= \$288{,}000$

The following is a review of the Valuation and Risk Models principles designed to address the AIM statements set forth by GARP®. This topic is also covered in:

Unexpected Loss

Topic 53

Exam Focus

This topic continues the discussion of expected loss from the previous topic and further examines the calculation of unexpected loss. We will derive an expression for unexpected loss equal to a fraction of adjusted exposure. As we will see, default and credit migration increase the unexpected loss of a risky asset (i.e., loan). For the exam, understand the relationship between expected loss and unexpected loss and know that economic capital is used to cover unexpected losses.

Expected and Unexpected Loss

AIM 53.1: Explain the objective for quantifying both expected and unexpected loss.

AIM 53.2: Describe factors contributing to expected and unexpected loss.

As seen in the previous topic, **expected loss** is defined as the anticipated deterioration in the value of a risky asset that the bank has taken onto its balance sheet. Recall that expected loss is a function of adjusted exposure (AE), loss given default (LGD), and expected default frequency (EDF).

$$EL = AE \times LGD \times EDF$$

This equation describes the average behavior of the risky asset. Over time, the value of the asset will fluctuate above and below its average level. At maturity, in most cases the asset will not have defaulted; however, a fraction of the time default will occur bringing a significant decrease in value. The EL measure does not capture the variation in the risky asset's value. This variation is referred to as the **unexpected loss** which we will examine further throughout this topic.

The unanticipated loss on the risky asset can arise from the incidence of default or credit migration. Default is a positive probability event for even the safest of borrowers. Banks can estimate the likelihood of default using historical data, the method employed by the rating agencies. On the other hand, default can be estimated using models based on the "option" view of the firm such as the Merton model (which will be discussed at Part II of the FRM program). This approach views the firm as holding a call option with a strike price equal to the value of the outstanding debt. If the value of the firm is less than the value of its debt obligations, the firm will default.

Credit migration denotes the possible deterioration in creditworthiness of the borrower. While a shift in migration may not result in immediate default, the probability of such an event increases. It is also possible for the reverse to occur, that is, the credit quality of the obligor improves over time.

Computing Unexpected Loss

AIM 53.3: Define, calculate and interpret the unexpected loss of an asset.

The observation that the unexpected loss represents the variability of potential losses can be modeled using the typical definition of standard deviation. If UL_H denotes the unexpected loss at the horizon for asset value V_H, then:

$$UL_H \equiv \sqrt{\text{var}(V_H)}$$

For the equations to follow, the subscript H will be dropped but be aware that we are focused on the horizon date, H. After some algebra, we derive the following expression:

$$UL = AE \times \sqrt{EDF \times \sigma^2{}_{LGD} + LGD^2 \times \sigma^2{}_{EDF}}$$

Since we assume a two-state model, the variance of EDF is simply the variance of a binomial random variable: $\sigma^2{}_{EDF} = EDF \times (1 - EDF)$. Further note, the AE term explicitly recognizes that only the risky portion of the asset is subject to default.

Professor's Note: Do not lose sight of the big picture here. We are merely applying the basic definition for standard deviation based on the terminal value of the risky asset on the horizon date.

It is also worthwhile to examine the multiplier (square root term) in more detail. Notice that each term is at most equal to one so the UL is a fraction of the adjusted exposure. In addition, in the extreme case where the default ($\sigma^2{}_{EDF} = 0$) and recovery ($\sigma^2{}_{LGD} = 0$) are known with certainty, the unexpected loss equals zero which confirms that the EL is constant and also known with certainty.

Example: Computing unexpected loss

Suppose XYZ bank has booked a loan with the following characteristics: total commitment of \$2,000,000 of which \$1,200,000 is currently outstanding. The bank has assessed an internal credit rating equivalent to a 1% default probability over the next year. Draw down upon default is assumed to be 75%. The bank has additionally estimated a 40% loss given default. The standard deviation of EDF and LGD is 5% and 30%, respectively. **Calculate** the unexpected loss for XYZ bank.

Answer:

We can calculate the expected and unexpected loss as follows.

$$EL = AE \times EDF \times LGD$$

$$\begin{aligned}\text{Adjusted exposure} &= OS + (COM - OS) \times UGD \\ &= \$1{,}200{,}000 + (\$800{,}000 \times 0.75) \\ &= \$1{,}800{,}000\end{aligned}$$

$$\begin{aligned}EL &= \$1{,}800{,}000 \times 0.01 \times 0.40 \\ &= \$7{,}200\end{aligned}$$

$$UL = AE \times \sqrt{EDF \times \sigma^2{}_{LGD} + LGD^2 \times \sigma^2{}_{EDF}}$$

$$UL = \$1{,}800{,}000 \times \sqrt{0.01 \times 0.3^2 + 0.4^2 \times 0.05^2} = \$64{,}900$$

The unexpected loss represents 3.61% of adjusted exposure ($64,900 / $1,800,000).

Economic Capital

AIM 53.4: Explain the relationship between economic capital, expected loss and unexpected loss.

The best estimate of the devaluation of the risky asset is the expected loss. However, as the previous example clearly illustrates, the unexpected loss can exceed the expected loss by a wide margin. If the bank holds inadequate reserves, there is a possibility that the bank will become impaired. Therefore, it is imperative that the bank hold capital reserves (i.e., **economic capital**) to buffer itself from unexpected losses so that it can absorb large losses and continue to operate.

Banks set aside credit reserves in preparation for expected losses. However, for unexpected losses, banks need to estimate the excess capital reserves needed to cover any unexpected losses. The excess capital needed to match the bank's estimate of unexpected loss is referred to as economic capital.

Key Concepts

1. Expected loss is the average decrease in value of the risky asset on the bank's balance sheet.
2. The distribution of losses is highly skewed; the risky asset will rarely default but if it does, the loss will be large. Hence, the expected loss level does not capture the wide potential variation in losses.
3. Unexpected loss has a natural interpretation as the standard deviation of loan loss.
4. Credit migration and default reduce the expected return on the risky bank asset.
5. Economic capital is the buffer the bank must keep on reserve to prevent insolvency. Note that the level of economic capital is significantly greater than the expected loss to guard against the possibility of losses that exceed the average loss.
6. Unexpected loss is calculated as $UL = AE \times \sqrt{EDF \times \sigma^2{}_{LGD} + LGD^2 \times \sigma^2{}_{EDF}}$.
7. Unexpected loss is a fraction of adjusted exposure and directly related to probability of default and loss given default.

Concept Checkers

1. Expected loss and unexpected loss:
   A. are measured over different time periods.
   B. implicitly incorporate recovery rates in their computations.
   C. are statistically independent.
   D. are inverse functions.

2. The unexpected loss will equal adjusted exposure:
   A. if probability of default = 0.
   B. if recovery rate = 100%.
   C. if there is maximum draw down.
   D. None of the above.

3. Suppose XYZ bank has booked a loan with the following characteristics: total commitment of $2,000,000 of which $1,200,000 is currently outstanding. The bank has assessed an internal credit rating equivalent to a 1% default probability over the next year. Draw down upon default is assumed to be 75%. The bank has additionally estimated a 40% recovery rate. The standard deviation of EDF and LGD is 5% and 30%, respectively. Unexpected loss for XYZ bank is closest to:
   A. $62,450.
   B. $38,200.
   C. $64,900.
   D. $76,400.

4. Decreasing the recovery rate will do which of the following to expected loss and unexpected loss?
   A. Increase both EL and UL.
   B. Increase EL and decrease UL.
   C. Decrease EL and increase UL.
   D. Decrease both EL and UL.

5. Decreasing the recovery rate will do which of the following to expected loss, adjusted exposure, and unexpected loss?
   A. Increase both EL and AE.
   B. Increase EL and leave AE unchanged.
   C. Decrease AE and increase UL.
   D. Decrease both AE and UL.

Concept Checker Answers

1. B Expected loss and unexpected loss are calculated over the same time period. Unexpected loss is dependent on expected loss in the standard deviation calculation. EL and UL cannot be inverses of each other.

2. D Examination of $UL = AE \times \sqrt{EDF \times \sigma^2_{LGD} + LGD^2 \times \sigma^2_{EDF}}$ indicates that neither A, B, or C could be true. In order for UL to be equal to AE, the expression under the square root would have to be equal to 1.

3. D The key to this problem is that the recovery rate is given as 40% which implies the LGD is 60%.

   We can calculate the expected and unexpected loss as follows:

   EL = AE × EDF × LGD

   Adjusted exposure = OS + (COM – OS) × UGD
   = \$1,200,000 + (\$800,000 × 0.75)
   = \$1,800,000

   EL = \$1,800,000 × 0.01 × 0.60
   = \$10,800

   $$UL = AE \times \sqrt{EDF \times \sigma^2_{LGD} + LGD^2 \times \sigma^2_{EDF}}$$

   $$UL = \$1,800,000 \times \sqrt{0.01 \times 0.3^2 + 0.6^2 \times 0.05^2} = \$76,368$$

4. A Reducing the recovery rate increases the expected loss. It will also increase the variability around the expected loss level, increasing standard deviation (unexpected loss).

5. B Adjusted exposure is unaffected by changes in recovery rates (i.e., loss given default). Recall the formula AE = OS + (fraction of commitment) × (usage given default).

Challenge Problems

1. Consider the following three bonds that all have par values of $100,000.

    I. A 10-year zero coupon bond priced at 48.20.
    II. A 5-year 8% semiannual-pay bond priced with a YTM of 8%.
    III. A 5-year 9% semiannual-pay bond priced with a YTM of 8%.

    Rank the three bonds in terms of how important reinvestment income is to an investor who wishes to realize the stated YTM of the bond at purchase by holding it to maturity.
    A. III, II, I.
    B. I, II, III.
    C. II, III, I.
    D. I, III, II.

Use the following information to answer Questions 2 through 6.

A bond dealer provides the following selected information on a portfolio of fixed-income securities.

Par Value	*Mkt. Price*	*Coupon*	*Modified Duration*	*Effective Duration*	*Effective Convexity*
$2 million	100	6.5%	8	8	308
$3 million	93	5.5%	6	1	100
$1 million	95	7%	8.5	8.5	260
$4 million	103	8%	9	5	–70

2. What is the effective duration for the portfolio?
    A. 4.81.
    B. 5.63.
    C. 7.17.
    D. 7.88.

3. What is the price value of a basis point for this portfolio?
    A. $5,551.18.
    B. $7,026.60.
    C. $3,234.08.
    D. $4,742.66.

4. Which bonds likely have no embedded options? (identify bonds by coupon)
    A. 6.5% and 8.0% bonds.
    B. 6.5% and 7.0% bonds.
    C. 5.5% and 7.0% bonds.
    D. 5.5% and 8.0% bonds.

5. Which bond is likely currently callable?
   A. 6.5% bond.
   B. 5.5% bond.
   C. 7.0% bond.
   D. 8.0% bond.

6. What is the approximate price change for the 7% bond if its yield to maturity increases by 25 basis points?
   A. –$19,415.63.
   B. –$17,864.11.
   C. –$20,181.85.
   D. –$16,748.53.

7. All things being equal, which of the following statements is(are) correct regarding deep in-the-money call options on a fixed income security?
   I. The rho is zero or close to zero.
   II. The vega is zero or close to zero.

   A. I only.
   B. II only.
   C. Both I and II.
   D. Neither I nor II.

8. The current stock price of Heart, Inc., is $80. Call and put options with exercise prices of $50 and 15 days to maturity are currently trading. Which of these scenarios is most likely to occur if the stock price falls by $1?

	Call value	Put value
A.	Decrease by $0.94	Increase by $0.08
B.	Decrease by $0.76	Increase by $0.96
C.	Decrease by $0.07	Increase by $0.89
D.	Decrease by $0.76	Increase by $0.89

9. A put option with an exercise price of $45 is trading for $3.50. The current stock price is $45. What is the most likely effect on the option's delta and gamma if the stock price increases to $50?
   A. Both delta and gamma will increase.
   B. Both delta and gamma will decrease.
   C. One will increase and the other will decrease.
   D. Both delta and gamma will stay the same.

10. From the Black-Scholes-Merton model, $N(d_1) = 0.42$ for a 3-month call option on Panorama Electronics common stock. If the stock price falls by $1.00, the price of the call option will:
   A. decrease by less than the increase in the price of the put option.
   B. increase by more than the decrease in the price of the put option.
   C. decrease by the same amount as the increase in the price of the put option.
   D. increase by more than the increase in the price of the put option.

11. As a junior quantitative analyst, you have been assigned to research coherent risk measures. Which of the following properties of coherent risk measures explicitly takes into the account the diversification benefits of holding assets in a portfolio with less-than-perfect correlation of returns?
    A. Monotonicity.
    B. Positive homogeneity.
    C. Subadditivity.
    D. Translation invariance.

12. As the operational risk manager at a large oil company, you are looking to use financing instruments to hedge the firm's operational risk. Which of the following would you not use to hedge the risk of an event occurring outside of the control of the firm?

    I. Indemnified notes.
    II. Indexed notes.

    A. I only.
    B. II only.
    C. Both I and II.
    D. Neither I nor II.

13. As a risk manager, you have been assigned to measure the firm's operating risk. Your supervisor wants you to focus on models that can diagnose weaknesses in specific operational procedures and suggest corrections (in other words, models that are forward-looking in the sense that they go beyond the use of historical data). Which of the following models would be most appropriate for you to use?

    I. Connectivity models.
    II. Multifactor models.

    A. I only.
    B. II only.
    C. Both I and II.
    D. Neither I nor II.

14. Scenario analysis involves estimating portfolio value from extreme movements in model inputs. Therefore, as a risk manager, you are considering the use of scenario analysis to complement your existing use of sensitivity analysis. Which of the following types of scenario analysis explicitly considers the correlation across risk factors?
    A. Conditional scenario method (of multidimensional scenario analysis).
    B. Factor push method (of multidimensional scenario analysis).
    C. Historical scenario analysis.
    D. Unidimensional scenario analysis.

15. As an associate risk manager at a bank, you are concerned about the various risks faced by the bank's securitization transactions. Which of the following risks refers to a bank having to hold onto assets for longer than planned and incurring financing costs as a result?
    A. Contingent risk.
    B. Funding liquidity risk.
    C. Pipeline risk.
    D. Wrong-way risk.

16. You are an associate at a rating agency reviewing a research report compiled by one of the new analysts. Which of the following statements in the report is correct?
    A. For a given rating category, default rates show statistically significant variation based on geographic location.
    B. For a given rating category, default rates show statistically significant variation based on industry.
    C. The cumulative default rate is generally more dramatic for a bond rated Baa3 than for a bond rated Ba1.
    D. The cumulative default rate is generally less dramatic for a bond rated BB than for a bond rated BBB.

17. Global Bank has made a loan with the following characteristics: total commitment of $5 million of which $4.1 million is currently outstanding. Global has assessed an internal credit rating equivalent to a 1.5% default probability over the next year. Drawdown upon default is assumed to be 65%. Global has additionally estimated a 35% loss given default. Calculate the expected loss.
    A. $21,525.
    B. $24,596.
    C. $26,250.
    D. $27,735.

Challenge Problem Answers

1. A Reinvestment income is most important to the investor with the 9% coupon bond, followed by the 8% coupon bond and the zero-coupon bond. In general, reinvestment risk increases with the coupon rate on a bond.

   (See Topic 38)

2. A Portfolio effective duration is the weighted average of the effective durations of the portfolio bonds.

   Numerators in weights are market values (par value × price as percent of par). Denominator is total market value of the portfolio.

   $$\frac{2}{9.86}(8)+\frac{2.79}{9.86}(1)+\frac{0.95}{9.86}(8.5)+\frac{4.12}{9.86}(5)=4.81 \text{ (weights are in millions)}$$

   (See Topic 39)

3. D Price value of a basis point can be calculated using effective duration for the portfolio and the portfolio's market value, together with a yield change of 0.01%. Convexity can be ignored for such a small change in yield.

   $4.81 \times 0.0001 \times 9{,}860{,}000 = \$4{,}742.66$

   (See Topic 39)

4. B The 6.5% and 7% coupon bonds likely have no embedded options. For both of these bonds, modified duration and effective duration are identical, which would be the case if they had no embedded options. It is possible that these bonds have options that are so far out-of-the-money that the bond prices act as if there is no embedded option.

   (See Topic 39)

5. D The 8% bond is likely callable. It is trading at a premium, its effective duration is less than modified duration, and it exhibits negative convexity. Remember, call price can be above par.

   (See Topic 39)

6. A Based on the effective duration and effective convexity of the 7% bond, the approximate price change is:

   $(-8.5 \times 0.0025) + (0.5 \times 260 \times 0.0025^2) \times 950{,}000 = -\$19{,}415.63$

   (See Topic 39)

7. **B** Statement II is correct. Both deep in-the-money and deep out-of-the-money call options have little sensitivity to changes in volatility (i.e., vega is close to zero).

   Statement I is incorrect because in-the-money calls are more sensitive to changes in rates than out-of-the-money options. Therefore, it is clear that rho is not zero or near zero in such situations because increases in interest rates cause larger increases for in-the-money call prices (versus out-of-the-money call prices).

   (See Topic 42)

8. **A** The call option is deep in-the-money and must have a delta close to one. The put option is deep out-of-the-money and will have a delta close to zero. Therefore, the value of the in-the-money call will decrease by close to $1 (e.g., $0.94), and the value of the out-of-the-money put will increase by a much smaller amount (e.g., $0.08). The call price will fall by more than the put price will increase.

   (See Topic 42)

9. **C** The put option is currently at-the-money since its exercise price is equal to the stock price of $45. As stock price increases, the put option's delta (which is less than zero) will increase toward zero, becoming less negative. The put option's gamma, which measures the rate of change in delta as the stock price changes, is at a maximum when the option is at-the-money. Therefore, as the option moves out-of-the-money, its gamma will fall.

   (See Topic 42)

10. **A** If $\Delta S = -\$1.00$, $\Delta C \approx 0.42 \times (-1.00) = -\$0.42$, and $\Delta P \approx (0.42 - 1) \times (-1.00) = \$0.58$.

    The call will decrease by less ($0.42) than the increase in the price of the put ($0.58).

    (See Topic 42)

11. **C** Subadditivity refers to the concept that the risk of a portfolio is at most equal to the risk of the assets within the portfolio. This suggests that portfolio risk would be less than the sum of the individual risks of the assets due to diversification.

    (See Topic 43)

12. **A** Statement I is correct because indemnified notes offer the firm relief from debt based on internal events (within the control of the firm), such as a large underwriting loss for an insurance company.

    Statement II is incorrect because indexed notes provide cash flows related to the value of an independent index, such as a weather index. Clearly, the cash flows are determined by an external event outside the control of the firm.

    (See Topic 46)

13. A Your boss wants you to use a bottom-up approach to analyzing risk instead of a top-down approach. Bottom-up models are able to diagnose weaknesses in specific operational procedures and suggest corrections. In that way, they are forward-looking, whereas top-down models rely exclusively on historical data and are backward-looking.

Connectivity models are an example of bottom-up models. Multifactor models are an example of top-down models.

(See Topic 46)

14. A The primary advantage to the conditional scenario method is the inclusion of correlations across risk factors. By focusing on changes in a subset of variables (holding the other variables constant), incorporation of the variance-covariance matrix is allowed.

(See Topic 47)

15. C Pipeline risk originates from market stress when a bank may not be able to complete the entire process of selling the securities to the public through the issue-special purpose entities. Consequently, pipeline risk arises because market conditions may force the bank to warehouse underlying assets for longer than planned and therefore, incurring financing costs.

(See Topic 48)

16. **B** Empirical evidence suggests that for a given rating category, default rates can vary from industry to industry to a statistically significant degree.

(See Topic 50)

17. **B** EL = adjusted exposure × expected default frequency × loss given default

Adjusted exposure = outstandings + (commitment – outstandings) × usage given default

Usage given default = funds drawn down on default

Adjusted exposure = \$4.1m + (\$5m – \$4.1m) × 0.65 = \$4.685m

EL = \$4.685m × 0.015 × 0.35

EL = \$24,596.25

(See Topic 52)

GARP FRM Practice Exam Questions

Valuation and Risk Models

Professor's Note: The following questions are from the 2008–2011 GARP FRM Practice Exams.

1. A bond with par value of USD 100 and 3 years to maturity pays 7% annual coupons. The spot rate curve is as follows:

Term	*Annual Spot Interest Rates*
1	6%
2	7%
3	8%

The value of the bond is closest to:
A. USD 95.25
B. USD 97.66
C. USD 99.25
D. USD 101.52

2. A 5-year corporate bond paying an annual coupon of 8% is sold at a price reflecting a yield-to-maturity of 6% per year. One year passes and the interest rates remain unchanged. Assuming a flat term structure and holding all other factors constant, the bond's price during this period will have:
A. increased
B. decreased
C. remained constant
D. cannot be determined with the data given

3. Consider a bond with par value of EUR 1,000, maturity in 3 years, and that pays a coupon of 5% annually. The spot rate curve is as follows:

Term	*Annual Spot Interest Rates*
1	6%
2	7%
3	8%

The value of the bond is closest to:
A. EUR 904
B. EUR 924
C. EUR 930
D. EUR 950

4. A risk manager for bank XYZ, Mark is considering writing a 6 month American put option on a non-dividend paying stock ABC. The current stock price is USD 50 and the strike price of the option is USD 52. In order to find the no-arbitrage price of the option, Mark uses a two-step binomial tree model. The stock price can go up or down by 20% each period. Mark's view is that the stock price has an 80% probability of going up each period and a 20% probability of going down. The risk-free rate is 12% per annum with continuous compounding. What is the risk-neutral probability of the stock price going up in a single step?
   A. 34.5%
   B. 57.6%
   C. 65.5%
   D. 80.0%

5. A non-dividend paying stock is currently trading at USD 25. You are looking to find a no-arbitrage price for a 1 year American call using a two-step binomial tree model for which the stock can go up or down by 25%. The risk free rate is 10% and you believe that there is an equal chance of the stock price going up or down. What is the risk-neutral probability of the stock price going down in a single step?
   A. 22.6%
   B. 39.8%
   C. 50.0%
   D. 68.3%

6. Assume that options on a non dividend paying stock with price of USD 100 have a time to expiry of half a year and a strike price of USD 110. The risk-free rate is 10%. Further, $N(d_1) = 0.457185$ and $N(d_2) = 0.374163$. Which of the following values is closest to the Black-Scholes values of these options?
   A. Value of American call option is USD 6.56 and of American put option is USD 12.0
   B. Value of American call option is USD 5.50 and of American put option is USD 12.0
   C. Value of American call option is USD 6.56 and of American put option is USD 10.0
   D. Value of American call option is USD 5.50 and of American put option is USD 10.0

7. Assume that options on a non dividend paying stock with price of USD 150 expire in a year and all have a strike price of USD 140. The risk-free rate is 8%. Which of the following values is closest to the Black-Scholes values of these options assuming $N(d_1) = 0.7327$ and $N(d_2) = 0.6164$?
   A. Value of American call option is USD 30.25 and of American put option is USD 9.48
   B. Value of American call option is USD 9.48 and of American put option is USD 30.25
   C. Value of American call option is USD 30.25 and of American put option is USD 0.00
   D. Value of American call option is USD 9.48 and of American put option is USD 0.00

8. An analyst is doing a study on the effect on option prices of changes in the price of the underlying asset. The analyst wants to find out when the deltas of calls and puts are most sensitive to changes in the price of the underlying. Assume that the options are European and that the Black-Scholes formula holds. An increase in the price of the underlying has the largest absolute value impact on delta for:
   A. deep in-the-money calls and deep out-of-the-money puts.
   B. deep in-the-money puts and calls.
   C. deep out-of-the-money puts and calls.
   D. at-the-money puts and calls.

9. A trader in your bank has bought 200 call option contracts each on 100 shares of General Motors with time to maturity of 60 days at USD 2.10. The delta of the option on one share is 0.50. As a risk manager, what action must you take on the underlying stock in order to hedge the option exposure and keep it delta neutral?
   A. Buy 10,000 shares of General Motors.
   B. Sell 10,000 shares of General Motors.
   C. Buy 1,000 shares of General Motors.
   D. Sell 1,000 shares of General Motors.

10. Which of the following portfolios would have the highest vega assuming all options involved are of the same strikes and maturities?
   A. Long a call
   B. Short a put
   C. Long a put and long a call
   D. A short of the underlying, a short in a put, and a long in a call

11. Which of the following statements is correct?
   I. The rho of a call option changes with the passage of time and tends to approach zero as expiration approaches, but this is not true for the rho of put options.
   II. Theta is always negative for long calls and long puts and positive for short calls and short puts.

   A. I only
   B. II only
   C. I and II
   D. Neither

12. Which of the following statements is incorrect, given the following one-year rating transition matrix?

From/To (%)	*AAA*	*AA*	*A*	*BBB*	*BB*	*B*	*CCC/C*	*D*	*Non Rated*
AAA	87.44	7.37	0.46	0.09	0.06	0.00	0.00	0.00	4.59
AA	0.60	86.65	7.78	0.58	0.06	0.11	0.02	0.01	4.21
A	0.05	2.05	86.96	5.50	0.43	0.16	0.03	0.04	4.79
BBB	0.02	0.21	3.85	84.13	4.39	0.77	0.19	0.29	6.14
BB	0.04	0.08	0.33	5.27	75.73	7.36	0.94	1.20	9.06
B	0.00	0.07	0.20	0.28	5.21	72.95	4.23	5.71	11.36
CCC/C	0.08	0.00	0.31	0.39	1.31	9.74	46.83	28.83	12.52

A. BBB loans have a 4.08% chance of being upgraded in one year.
B. BB loans have a 75.73% chance of staying at BB for one year.
C. BBB loans have an 88.21% chance of being upgraded in one year.
D. BB loans have a 5.72% chance of being upgraded in one year.

13. Which of the following statements is incorrect, given the following one-year rating transition matrix?

From/To (%)	*AAA*	*AA*	*A*	*BBB*	*BB*	*B*	*CCC/C*	*D*	*Non Rated*
AAA	87.44	7.37	0.46	0.09	0.06	0.00	0.00	0.00	4.59
AA	0.60	86.65	7.78	0.58	0.06	0.11	0.02	0.01	4.21
A	0.05	2.05	86.96	5.50	0.43	0.16	0.03	0.04	4.79
BBB	0.02	0.21	3.85	84.13	4.39	0.77	0.19	0.29	6.14
BB	0.04	0.08	0.33	5.27	75.73	7.36	0.94	1.20	9.06
B	0.00	0.07	0.20	0.28	5.21	72.95	4.23	5.71	11.36
CCC/C	0.08	0.00	0.31	0.39	1.31	9.74	46.83	28.83	12.52

A. 'AAA' loans have 0% chance of ever defaulting.
B. 'AA' loans have a 86.65% chance of staying at AA for one year.
C. 'A' loans have a 13.04% chance of receiving a ratings change.
D. 'BBB' loans have a 4.08% chance of being upgraded in one year.

14. You have been asked by the Chief Risk Officer of your bank to determine how much should be set aside as a loan-loss reserve for a 1-year horizon on a USD 100 million line of credit that has been extended to a large corporate borrower. Of the original balance, USD 20 million has already been drawn and due to deteriorating economic conditions the bank is concerned that the borrower might find itself in a liquidity crisis causing it to draw on the remaining commitment and default. Given the following information from the bank's internal credit risk models what is an appropriate loan loss reserve to cover this eventuality?

- 1-year default probability = 0.35%
- Drawdown given default = 80%
- Loss given default = 60%

A. USD 210,000
B. USD 176,400
C. USD 140,000
D. USD 117,600

15. The following table gives the prices of two out of three US Treasury notes for settlement on August 30, 2008. All three notes will mature exactly one year later on August 30, 2009. Assume annual coupon payments and that all three bonds have the same coupon payment date.

Coupon	*Price*
2 7/8	98.40
4 1/2	?
6 1/4	101.30

Approximately what would be the price of the 4 1/2 US Treasury note?
A. 99.20
B. 99.40
C. 99.80
D. 100.20

16. The table below gives the closing prices and yields of a particular liquid bond over the past few days.

Day	*Price*	*Yield*
Monday	106.3	4.25%
Tuesday	105.8	4.20%
Wednesday	106.1	4.23%

What is the approximate duration of the bond?
A. 18.8
B. 9.4
C. 4.7
D. 1.9

17. A newly issued non-callable, fixed-rate bond with 30-year maturity carries a coupon rate of 5.5% and trades at par. Its duration is 15.33 years and its convexity is 321.03. Which of the following statements about this bond is true?
    A. If the bond were to start trading at a discount, its duration would decrease.
    B. If the bond were to start trading at a premium, its duration would decrease.
    C. If the bond were to start trading at a discount, its duration would not change.
    D. If the bond were to remain at par, its duration would increase as the bond aged.

18. Bonds issued by the XYZ Corp. are currently callable at par value and trade close to par. The bonds mature in 8 years and have a coupon of 8%. The yield on the XYZ bonds is 175 basis points over 8-year US Treasury securities, and the Treasury spot yield curve has a normal, rising shape. If the yield on bonds comparable to the XYZ bond decreases sharply, the XYZ bonds will most likely exhibit:
    A. negative convexity
    B. increasing modified duration
    C. increasing effective duration
    D. positive convexity

19. Currently, shares of ABC Corp. trade at USD 100. The monthly risk neutral probability of the price increasing by USD 10 is 30%, and the probability of the price decreasing by USD 10 is 70%.What are the mean and standard deviation of the price after 2 months if price changes on consecutive months are independent?

	Mean	Standard Deviation
A.	70	11.32
B.	70	12.96
C.	92	11.32
D.	92	12.96

20. Mr. Black has been asked by a client to write a large put option on the S&P 500 index. The option has an exercise price and a maturity that are not available for options traded on exchanges. He, therefore, has to hedge the position dynamically. Which of the following statements about the risk of his position are not correct?
    A. He can make his portfolio delta neutral by shorting index futures contracts.
    B. There is a short position in an S&P 500 futures contract that will make his portfolio insensitive to both small and large moves in the S&P 500.
    C. A long position in a traded option on the S&P 500 will help hedge the volatility risk of the option he has written.
    D. To make his hedged portfolio gamma neutral, he needs to take positions in options as well as futures.

21. The current share price and daily volatility of a stock are USD 10 and 2%, respectively. Using the delta-normal approximation, the 95% VaR on a long at-the-money call on this stock over a one-day holding period is:
    A. USD 0.1645
    B. USD 0.3290
    C. USD 1.645
    D. USD 16.45

22. One of the traders whose risk you monitor put on a carry trade where he borrows in yen and invests in some emerging market bonds whose performance is independent of yen. Which of the following risks should you not worry about?
    A. Unexpected devaluation of the yen.
    B. A currency crisis in one of the emerging markets the trader invests in.
    C. Unexpected downgrading of the sovereign rating of a country in which the trader invests.
    D. Possible contagion to emerging markets of a credit crisis in a major country.

23. The price of a 3-year zero coupon government bond is 85.16. The price of a similar 4-year bond is 79.81. What is the one-year implied forward rate from year 3 to year 4?
    A. 5.4%
    B. 5.5%
    C. 5.8%
    D. 6.7%

24. A portfolio manager has a bond position worth USD 100 million. The position has a modified duration of 8 years and a convexity of 150 years. Assume that the term structure is flat. By how much does the value of the position change if interest rates increase by 25 basis points?
    A. USD –1,953,125
    B. USD –1,906,250
    C. USD –2,046,875
    D. USD –2,187,500

25. Initially, the call option on Big Kahuna Inc. with 90-days to maturity trades at USD 1.40. The option has a delta of 0.5739. A dealer sells 200 call option contracts and to delta-hedge the position, the dealer purchases 11,478 shares of the stock at the current market price of USD 100 per share. The following day, the prices of both the stock and the call option increase. Consequently, delta increases to 0.7040. To maintain the delta hedge, the dealer should:
    A. purchase 2,602 shares.
    B. sell 2,602 shares.
    C. purchase 1,493 shares.
    D. sell 1,493 shares.

26. Consider the following three methods of estimating the P&L of a bullet bond: full repricing, duration (PV01), and duration plus convexity. Ranking the estimated P&L impact of a large negative yield shock from the lowest P&L impact to the highest P&L impact, what is the ranking of the methods to estimate the P&L impact?
    A. Duration plus convexity, duration, full repricing
    B. Full repricing, duration plus convexity, duration
    C. Duration, duration plus convexity, full repricing
    D. Duration, full repricing, duration plus convexity

27. Suppose you are given the following information about the operational risk losses at your bank.

Frequency Distribution		*Severity Distribution*	
Probability	*Frequency*	*Probability*	*Severity*
0.5	0	0.6	USD 1,000
0.3	1	0.3	USD 10,000
0.2	2	0.1	USD 100,000

What is the estimate of the VaR at the 95% confidence level, assuming that the frequency and severity distributions are independent?
A. USD 200,000
B. USD 110,000
C. USD 100,000
D. USD 101,000

28. Determine the percentage of the following portfolio that is investment grade:

Moody's Rating	*Percentage of Portfolio*
Aa2	25%
A3	10%
Caa1	2%
Baa3	10%
Ba1	5%
D	3%
Aaa	10%
A1	15%
Baa1	10%
Aa3	10%

A. 70%
B. 80%
C. 90%
D. 95%

29. Consider the following one-period transition matrix:

Initial Period State	Next Period State A	B	Default
A	95%	5%	0%
B	10%	80%	10%
Default	0%	0%	100%

If a company is originally in State A, what is the probability that the company will have defaulted strictly before the fourth transition period from now?
A. 0.875%
B. 0.500%
C. 1.375%
D. 1.875%

30. The following table from Fitch Ratings shows the number of rated issuers migrating between two ratings categories during one year. Based on this information, what is the probability that an issue with a rating of A at the beginning of the year will be downgraded by the end of the year?

		Year 1 Rating					
		AAA	AA	A	BBB	Default	Total
Year 0 Rating	AAA	45	4	2	0	0	51
	AA	3	30	4	3	2	42
	A	2	5	40	2	3	52
	BBB	0	1	2	30	1	34
	Default	0	0	0	0	0	0

A. 13.46%
B. 13.44%
C. 9.62%
D. 3.85%

31. Beta Bank owns a portfolio of 10 AA-rated bonds with a total value of 200 million USD. The one-year probability of default for each issuer is 5% and the recovery rate for each issue equals 40%. The one-year expected loss of the portfolio is:
A. USD 4.0 million
B. USD 5.0 million
C. USD 6.0 million
D. USD 8.0 million

32. Given the following portfolio of bonds:

Bond	*Price*	*Par amount held (in USD million)*	*Modified duration*
A	101.43	3	2.36
B	84.89	5	4.13
C	121.87	8	6.27

What is the value of the portfolio's DV01 (Dollar value of 1 basis point)?
A. 8,019
B. 8,294
C. 8,584
D. 8,813

33. Suppose a 20-year annual coupon bond has a DV01 of 0.14865. Also suppose a 12-year annual coupon bond, which will be used as the hedging instrument, has a DV01 of 0.09764. If the yield beta is 1.10, which of the following statements accurately describes the situation?
A. The hedging instrument is significantly more volatile than the position in the 20-year bond, and the hedge ratio is 1.67467.
B. The position in the 20-year bond is significantly more volatile than the hedging instrument, and the hedge ratio is 0.72253.
C. In order to have a perfectly hedged position, for every USD 1 of the 20-year bond, USD 1.67467 of the 12-year bond should be shorted.
D. In order to have a perfectly hedged position, for every USD 1 of the 20-year bond, USD 0.72253 of the 12-year bond should be shorted.

34. Jeff is an arbitrage trader, and he wants to calculate the implied dividend yield on a stock while looking at the over-the-counter price of a 5-year put and call (both European-style) on that same stock. He has the following data:
    - Initial stock price = USD 85
    - Strike price = USD 90
    - Continuous risk-free rate = 5%
    - Underlying stock volatility = unknown
    - Call price = USD 10
    - Put price = USD 15

    What is the continuous implied dividend yield of that stock?
    A. 2.48%
    B. 4.69%
    C. 5.34%
    D. 7.71%

35. You are given the following information about a call option:
    - Time to maturity = 2 years
    - Continuous risk-free rate = 4%
    - Continuous dividend yield = 1%
    - N(d1) = 0.64

    Calculate the delta of this option.
    A. –0.64
    B. 0.36
    C. 0.63
    D. 0.64

36. Suppose an existing short option position is delta-neutral, but has a gamma of negative 600. Also assume that there exists a traded option with a delta of 0.75 and a gamma of 1.50. In order to maintain the position gamma-neutral and delta-neutral, which of the following is the appropriate strategy?
    A. Buy 400 options and sell 300 shares of the underlying asset.
    B. Buy 300 options and sell 400 shares of the underlying asset.
    C. Sell 400 options and buy 300 shares of the underlying asset.
    D. Sell 300 options and buy 400 shares of the underlying asset.

37. You want to implement a portfolio insurance strategy using index futures designed to protect the value of a portfolio of stocks not paying any dividends. Assuming the value of your stock portfolio decreases, which strategy would you implement to protect your portfolio?
    A. Buy an amount of index futures equivalent to the change in the call delta × original portfolio value.
    B. Sell an amount of index futures equivalent to the change in the call delta × original portfolio value.
    C. Buy an amount of index futures equivalent to the change in the put delta × original portfolio value.
    D. Sell an amount of index futures equivalent to the change in the put delta × original portfolio value.

38. The dividend yield of an asset is 10% per annum. What is the delta of a long forward contract on the asset with 6 months to maturity?
    A. 0.95
    B. 1.00
    C. 1.05
    D. Cannot be determined without further information

39. Which of the following statements is false?
    A. European-styled call and put options are most affected by changes in vega when they are at-the-money.
    B. The delta of a European-styled put option on an underlying stock would move towards zero as the price of the underlying stock rises.
    C. The gamma of an at-the-money European-styled option tends to increase as the remaining maturity of the option decreases.
    D. Compared to an at-the-money European-styled call option, an out-of-the money European option with the same strike price and remaining maturity would have a greater negative value for theta.

40. As the newly appointed head of operational risk for a large international bank, you must evaluate the company's current approach to estimating the firm-wide operational loss distribution. The bank's current approach is a bottoms-up process in which for each trading desk the operational loss severity distribution is estimated by fitting historical loss magnitude data to a Weibull distribution and the operational loss frequency distribution is estimated by fitting historical loss timing data to a Poisson distribution. Each trading desk's operational loss distribution is then estimated by aggregating the frequency and severity distributions using convolution. Finally, the firm-wide operational loss distribution is estimated using a copula function generated through Monte Carlo simulation. In evaluating this process, which of the following assumptions implied by the current approach will require further investigation?

    I. The independence of operational loss events of each particular trading desk.
    II. The independence of the frequency of operational loss events and the severity of operational loss events of each particular trading desk.
    III. The independence of operational loss events between trading desks.
    IV. The reliability and sufficiency of historical loss data for each trading desk.

    A. I, II, III, and IV
    B. I, II, and IV
    C. II, III, and IV
    D. I and III

41. Which of the following is true about stress testing?
    A. It is used to evaluate the potential impact on portfolio values of unlikely, although plausible, events or movements in a set of financial variables.
    B. It is a risk-management tool that directly compares predicted results to observed actual results. Predicted values are also compared with historical data.
    C. Both 'a' and 'b' above are true.
    D. None of the above are true.

42. Which of the following is not a true statement about internal credit ratings?
    A. The "at-the-point-in-time" approach makes heavy use of econometric modeling that relates current financial variables to estimated default risk.
    B. The "through-the-cycle" approach is forward-looking and attempts to incorporate future economic scenarios into current default risk estimates.
    C. "At-the-point-in-time" credit scores volatility is much higher than "through-the-cycle" score volatility.
    D. A sound internal system uses at-the-point-in-time scoring for small-to-medium-sized companies and private firms and through-the-cycle scoring for large firms.

43. Given the 1-year transition matrix below, what is the probability that a company that is currently B rated will default over a given 2-year period?

Initial Period State	Next Period State		
	A	B	Default
A	85%	10%	5%
B	10%	80%	10%

    A. 10.0%
    B. 18.0%
    C. 18.5%
    D. 20.0%

GARP FRM Practice Exam Answers

Question from the 2011 FRM practice exam.

1. **B** USD 97.66

   Using spot rates, the value of the bond is 7 / (1.06) + 7 / [(1.07)^2] + 107 / [(1.08)^3] = 97.66

   (See Topic 36)

Question from the 2011 FRM practice exam.

2. **B** Decreased

   Since yield-to-maturity < coupon, the bond is sold at a premium. As time passes, the bond price will move towards par. Hence the price will decrease.

   (See Topic 37)

Question from the 2011 FRM practice exam.

3. **B** EUR 924

   Using spot rates, the value of the bond is:

   50 / (1.06) + 50 / [(1.07)^2J + 1050 / [(1.08)^3J = 924.37

   (See Topic 37)

Question from the 2011 FRM practice exam.

4. **B** 57.6%

   $P_{up} = (e^{r\Delta t} - d) / (u - d) = (e^{0.12 * 3 / 12} - 0.8) / (1.2 - 0.8) = 57.61\%$

   (See Topic 40)

Question from the 2011 FRM practice exam.

5. **B** 39.8%

   $P_{up} = (e^{r\Delta t} - d) / (u - d) = (e^{0.10 * 6 / 12} - 0.75) / (1.25 - 0.75) = 60.25\%$

   $P_{down} = 1 - P_{up} = 39.75\%$

   (See Topic 40)

Question from the 2011 FRM practice exam.

6. A Value of American call option is USD 6.56 and of American put option is USD 12.0

With the given data, the value of a European call option is USD 6.56 and the value of a European put option is USD 11.20. We know that American options are never less than corresponding European option in valuation. Also, the American call option price is exactly the same as the European call option price under the usual Black-Scholes world with no dividend. Thus only 'a' is the correct option.

(See Topic 41)

Question from the 2011 FRM practice exam.

7. A Value of American call option is USD 30.25 and of American put option is USD 9.48

With the given data the value of European call option is USD 30.25 and value of European put option is USD 9.48. We know that American options are never less than corresponding European option in valuation. Also, the American call option price is exactly the same as the European call option price under the usual Black-Scholes world with no dividend. Thus only 'a' is the correct option.

(See Topic 41)

Question from the 2011 FRM practice exam.

8. D At-the-money puts and calls.

A. Incorrect. When calls are deep in-the-money and puts are deep out-of-the-money, deltas are NOT most sensitive to changes in the underlying asset.
B. Incorrect. When both calls and puts are deep in-the-money, deltas are NOT most sensitive to changes in the underlying asset.
C. Incorrect. When both calls and puts are deep out-of-the-money, deltas are NOT most sensitive to changes in the underlying asset.
D. Correct. When both calls and puts are at-the-money, deltas are most sensitive to changes in the underlying asset. (Gammas are largest when options are at-the-money.)

(See Topic 42)

Question from the 2011 FRM practice exam.

9. A Buy 10,000 shares of General Motors.

Number of Calls = 200 Contracts * 100 = 20,000 Calls

Hedged by 20000 * .50 = 10000 shares

So, one needs to buy 10,000 shares in order to keep the position delta neutral.

(See Topic 42)

Question from the 2011 FRM practice exam.

10. **C** Long a put and long a call

    a and b are standard call/put, c is a straddle, d is a collar. A collar limits exposure to volatility, while a straddle increases this exposure. Vega is the sensitivity of a portfolio to volatility.

    (See Topic 42)

Question from the 2011 FRM practice exam.

11. **B** II only

    Statement I is false—rho of a call and a put will change, with expiration of time and it tends to approach zero as expiration approaches.

    (See Topic 42)

Question from the 2011 FRM practice exam.

12. **C** BBB loans have an 88.21% chance of being upgraded in one year.

    A. Incorrect. The chance of BBB loans being upgraded over 1 year is 4.08% (0.02 + 0.21 + 3.85).
    B. Incorrect. The chance of BB loans staying at the same rate over 1 year is 75.73%.
    C. Correct. 88.21% represents the chance of BBB loans staying at BBB or being upgraded over 1 year.
    D. Incorrect. The chance of BB loans being downgraded over 1 year is 5.72% (0.04 + 0.08 + 0.33 + 5.27).

    (See Topic 50)

Question from the 2011 FRM practice exam.

13. A 'AAA' loans have 0% chance of ever defaulting.

    AAA loans can default eventually, through consecutive downgrading, even though they are calculated to not default in one year.

    AA → AA is 86.65%

    A → A is 86.96%

    BBB → AAA/AA/A (sum) = 4.08%

    (See Topic 50)

Question from the 2011 FRM practice exam.

14. **B** USD 176,400

    The risky portion of the asset value at the horizon is Outstanding + (Commitment – Outstanding) * Drawdown Given Default = USD 20,000,000 + (USD 100,000,000 – USD 20,000,000) * 0.80 = USD 84,000,000. This is the adjusted exposure on default (AE). The expected loss EL = AE * EDF * LGD, or USD 84,000,000 * 0.0035 * 0.6 = USD 176,400. This is the amount that the bank should set aside as a loss reserve.

    (See Topic 52)

Question from the 2010 FRM practice exam.

15. **C** 99.80

$2.875\% * x + 6.25\% * (1 - x) = 4.5\%; X = 52\%$

The portfolio that has cash flows identical to the 4 1/2 bond consists of 52% of the 2 7/8 and 48% of the 6 1/4 bonds. As this portfolio has cash flows identical to the 4 1/2 bond, precluding arbitrage, the price of the portfolio should equal to 52% * 98.4 + 48% * 101.30 or 99.80.

(See Topic 36)

Question from the 2010 FRM practice exam.

16. **B** 9.4

The duration can be approximated from the price changes.

$(106.3 - 105.8) / 106.3 / .0005 = 9.4$
$(106.3 - 106.1) / 106.3 / .0002 = 9.4$

(See Topic 39)

Question from the 2010 FRM practice exam.

17. A If the bond were to start trading at a discount, its duration would decrease.

A. Correct. At higher interest rates, the bond/price relationship is closer to linear than it is when rates are low. So, the new duration would be lower than 15. Alternatively, one can think of duration as a weighted average of the times when cash flows are made, where the weights are the percentage of the total value of the bond. When rates rise, the present values associated with the later payments are relatively smaller and the duration falls.
B. Incorrect. It is the exact opposite of a, the correct answer.
C. Incorrect. It fails to recognize the logic stated in a.
D. Incorrect. Duration is mainly a function of duration and, all else constant, duration would decrease as the bond's maturity shortened.

(See Topic 39)

Question from the 2010 FRM practice exam.

18. A negative convexity

A. Correct. As yields in the market declines, the probability that the call option will get exercised increases. The issuer will not necessarily exercise the call option as soon as the market yield drops below the coupon rate. Yet the value of the embedded call option increases causing the price to reduce relative to an otherwise comparable option free bond. This is negative convexity.
B. Incorrect. Modified duration does not take into account the effect of embedded options.
C. Incorrect. As the interest rates decline, the call option becomes more valuable therefore effective duration may decrease because the expected cash flows can decrease.
D. Incorrect. When interest rates decline below the coupon rate, callable bonds show negative convexity.

(See Topic 39)

Question from the 2010 FRM practice exam.

19. D Mean: 92; Standard Deviation: 12.96

Develop a 2 step tree.

Mean = 9% (120) + 42% (100) + 49% (80) = 92
Variance = 9% $(120 - 92)^2$ + 42% $(100 - 92)^2$ + 49% $(80 - 92)^2$ = 168
Thus, standard deviation = 12.96

(See Topic 40)

Question from the 2010 FRM practice exam.

20. B There is a short position in an S&P 500 futures contract that will make his portfolio insensitive to both small and large moves in the S&P 500.

The short index futures makes the portfolio delta neutral. It does not help with large moves, though.

(See Topic 42)

Question from the 2010 FRM practice exam.

21. A USD 0.1645

This question requires candidates to know the formula for the delta-normal VaR approximation, and also to know that the delta of an at-the-money call is 0.5. VaR = 0.5 × 1.645 × 0.02 × 10 = 0.1645.

A. Correct.
B. Incorrect. Uses a delta of 1.
C. Incorrect. Confuses the decimal point.
D. Incorrect. Uses 2 instead of 2% for the volatility.

(See Topics 42 and 43)

Question from the 2010 FRM practice exam.

22. A Unexpected devaluation of the yen.

A devaluation would result in a gain to the trader because he is short yen.

(See Topic 51)

Question from the 2009 FRM practice exam.

23. **D** 6.7%

$$1+\text{Forward rate} = \frac{\text{Price of three year bond}}{\text{Price of four year bond}} = \frac{85.16}{79.81} = 1.067034$$

$$\text{Forward rate} = 0.067034 \text{ or } 6.7\%$$

A. Incorrect. This is B/C.
B. Incorrect. This is the return of the 3-year bond.
C. Incorrect. This is the return of the 4-year bond.

(See Topic 37)

Question from the 2009 FRM practice exam.

24. **A** USD –1,953,125

$$\Delta V = -D_{mod} \times \Delta y \times V + 0.5 \times \text{Convexity} \times \Delta y^2 \times V$$
$$\Delta V = -8 \times 0.0025 \times 100M + 0.5 \times 150 \times (0.0025)^2 \times 100M$$
$$\Delta V = -2M + 46{,}875$$
$$\Delta V = -1{,}953{,}125$$

B. Incorrect. Omits 0.5 from the second term.
C. Incorrect. Subtracts the second term.
D. Incorrect. Makes both mistakes.

(See Topic 39)

Question from the 2009 FRM practice exam.

25. A Purchase 2,602 shares.

Number of calls = 200 contracts × 100 = 20,000 calls.
Number of shares = (Number of calls) × (New delta – Old delta)
= 20,000 × (0.7040 – 0.5739)
= +2,602 shares

Positive sign indicates that the manager should purchase new shares.

B. Incorrect. The formula is incorrect, i.e. old delta minus new delta.
C. Incorrect. The number of shares (instead of number of calls) is used in the calculation.
D. Incorrect. As per explanation in 'C' above and sign error.

(See Topic 42)

Question from the 2009 FRM practice exam.

26. **C** Duration, duration plus convexity, full repricing

The price / yield line with yield on the x axis and price on the y axis is convex to the origin. The duration at any yield level is the tangent to that curve. Therefore, except at the exact point of tangency, duration will always underestimate the price change.

A. Incorrect. Duration will always underestimate price change for negative yield shocks.
B. Incorrect. Full repricing will never generate a smaller positive price change than duration because duration represents the point of tangency.
D. Incorrect. Full repricing will generate a higher price for a large negative yield change than will duration plus convexity.

(See Topic 44)

Question from the 2009 FRM practice exam.

27. **C** USD 100,000

The loss distribution is:

Total loss	*Probability*
0	0.5
1,000	0.18
2,000	0.072
10,000	0.09
11,000	0.072
20,000	0.018
100,000	0.03
101,000	0.024
110,000	0.012
200,000	0.002

The 95% VaR is 100,000. The other answers are from this distribution, but not corresponding to the 95% VaR.

(See Topic 46)

Question from the 2009 FRM practice exam.

28. **C** 90%

Non-investment grade assets are those rated below Baa3. Thus Caa1 with 2%, Ba1 with 5%, and D with 3%, or a total of 10% are non-investment grade. Thus the investment grade part should equal 90%.

(See Topic 49)

Question from the 2009 FRM practice exam.

29. **C** 1.375%

The easiest way to determine the answer would be to make this a square matrix including default in initial state. Then self-multiplying the matrix three times yields three-period transition matrix. We can also manually do the calculation;
After year 1 there is a 0% chance of default and 5% chance of being in state B.
After year 2 there is 95% * 5% + 80% * 5% chance of being in state B and 5% * 10% chance of default.
After year 3 there is a (95% * 5% + 80% * 5%) * 10% additional chance of default.

A. Incorrect. Only considers the third year transition from B to default.
B. Incorrect. Only considers the second year transition from B to default.
D. Incorrect. Mistakenly doubles the second year transition from B to default.

(See Topic 50)

Question from the 2009 FRM practice exam.

30. **C** 9.62%

Total Number of 'A' rated issuances = 52
Probability of 'A' rated issues downgraded to BBB (P_1) = 2 / 52 = 0.0385
Probability of 'A' rated issues downgraded to Default (P_2) = 3 / 52 = 0.0577
Probability of 'A' rated issues to be downgraded in one year = $P_1 + P_2$ = 0.0962 = 9.62%.

A. Incorrect. Is the number of upgrades from A (2 + 5) / 52 = 13.46%.
B. Incorrect. Is the number of downgrades to A 2 / 51 + 4 / 42 = 3.92% + 9.52% = 13.44%.
D. Incorrect. Is the number of downgrades to BBB 2 / 52 = 3.85%.

(See Topic 50)

Question from the 2009 FRM practice exam.

31. C USD 6.0 million

Expected Loss equals exposure multiplied by the risk of default and by the recovery rate, or E(L) = Exposure * PD * (1 – Recovery Rate).
E(L) = 200 million USD × 5% × 60% = 6 million USD.
Correlation amongst issuers does not matter for computing expected losses.

A. Incorrect. Incorrectly set E(L) = 200 * 0.05 * 0.4 = 4.0.
B. Incorrect. Incorrectly set E(L) = 200 * 0.05 * 0.5 = 5.0.
D. Incorrect. Incorrectly set E(L) = 200 * 0.05 * 0.8 = 8.0.

(See Topic 52)

Question from the 2008 FRM practice exam.

32. C 8,584

The portfolio dollar duration of a basis point (DV01)
= (portfolio modified duration × market value of portfolio)/10,000
The portfolio modified duration is obtained by taking the weighted average of the modified duration of the bonds in the portfolio.
Mathematically, it is as follows: $w_1D_1 + w_2D_2 + w_3D_3 + \ldots + w_kD_k$,
where w_i = market value of bond i/market value of the portfolio
D_i = modified duration of bond *i*
K = number of bonds of the portfolio.

Based on the above, the market values are as follows:
bond A = 101.43 × 3,000,000/100 = 3,042,900
bond B = 84.89 × 5,000,000/100 = 4,244,500
bond C = 121.87 × 8,000,000/100 = 9,749,600

Total market value of the portfolio = 3,042,900 + 4,244,500 + 9,749,600 = 17,037,000

Portfolio modified duration is calculated as follows:
(3,042,900/17,037,000)2.36 + (4,244,500/17,037,000)4.13 + (9,749,600/17,037,000)6.27
= (0.1786)2.36 + (0.2491)4.13 + (0.5723)6.27 = 0.4215 + 1.0289 + 3.5881 = 5.0385
Therefore, the portfolio dollar duration of a basis point (DV01) is obtained as follows:
(5.0385 × 17,037,000)/10,000 = 8,584

(See Topic 39)

Question from the 2008 FRM practice exam.

33. C In order to have a perfectly hedged position, for every USD 1 of the 20-year bond, USD 1.67467 of the 12-year bond should be shorted.

A. Incorrect. While the calculated hedge ratio is correct, its interpretation is incorrect.
B. Incorrect. Hedge ratio has been incorrectly calculated with the DV01 of hedging instrument in the numerator and DV01 of the position in the denominator (whereas it should be the other way).
C. Correct. Hedge Ratio = (0.14865 × 1.10) / 0.09764 = 1.674672. Interpretation in answer 'C' is accurate for hedge ratio.
D. Incorrect. Because the calculated hedge ratio is incorrect.

(See Topic 39)

Question from the 2008 FRM practice exam.

34. C 5.34%

We can use the Put-Call parity here to easily solve for the continuous dividend yield.
We have $C - P = S_0e^{-q*T} - Ke^{-r*T}$, so $10 - 15 = 85e^{-q*5} - 90e^{-0.05*5}$. Solving for q, we get 5.34%.

A. Incorrect. C and P were inverted in the formula.
B. Incorrect. C and P were inverted in the formula, and S and K were also inverted in the formula.
C. Correct. The above formula was used correctly, $C - P = S_0e^{-q*T} - Ke^{-r*T}$.
D. Incorrect. S and K were inverted in the formula.

(See Topic 41)

Question from the 2008 FRM practice exam.

35. **C** 0.63

The delta of a call option with a continuous dividend yield is given by the following formula: Delta= $N(d_1)*e^{-qT}$, where q is the continuous dividend yield, and T is the time to maturity. So, Delta = $0.64*e^{-0.01*2} = 0.63$.

A. Incorrect. The delta of a call is not equal to $-N(d_1)$.
B. Incorrect. The delta of a call is not equal to $1 - N(d_1)$.
C. Correct. The above formula was used correctly, $N(d_1)*e^{-qT}$.
D. Incorrect. The delta of a call with dividend yield is not equal to $N(d_1)$, the q was not used in the above formula.

(See Topic 42)

Question from the 2008 FRM practice exam.

36. A Buy 400 options and sell 300 shares of the underlying asset.

A. Correct. To gamma-hedge, we should buy 400 options (600/1.50). The additional options will alter delta-hedge, and to maintain delta-hedge position again, we should sell 300 shares (400 × 0.75) of the underlying position.
B. Incorrect. This strategy will neither delta-hedge nor gamma-hedge the position.
C. Incorrect. This strategy will gamma-hedge, but not delta-hedge the position.
D. Incorrect. This strategy will neither delta-hedge nor gamma-hedge the position.

(See Topic 42)

Question from the 2008 FRM practice exam.

37. **D** Sell an amount of index futures equivalent to the change in the put delta × original portfolio value.

A. Incorrect. For portfolio insurance strategy to work, index futures should be sold in an amount corresponding to the change in the put delta × original portfolio value.
B. Incorrect. For portfolio insurance strategy to work, index futures should be sold in an amount corresponding to the change in the put delta × original portfolio value.
C. Incorrect. For portfolio insurance strategy to work, index futures should be sold in an amount corresponding to the change in the put delta × original portfolio value.
D. Correct. Portfolio insurance strategy is accomplished by selling index futures contracts in an amount equivalent to the proportion of the portfolio dictated by the delta of the required put option. When a decrease in the value of the underlying portfolio occurs, the amount of additional index futures sold corresponds to the change in the put delta × original portfolio value.

(See Topic 42)

Question from the 2008 FRM practice exam.

38. A 0.95

Calculation:
The value of a long forward contract

$f = S_0\,e^{-qT} - K\,e^{-rT}$,

where S_0, q, T, K, and r are the asset price, dividend yield, time to maturity, delivery price, and risk-free rate, respectively.

It follows that the delta of the forward = e^{-qT}.
Given q = 10% and T = 1/2, we have delta = $e^{-10\%/2} = 0.95$

A. Correct. Shown from the calculations above.
B. Incorrect. Derived erroneously by not accounting for the dividend.
C. Incorrect. Derived erroneously by mixing up the sign of exponential.
D. Incorrect. It can be determined with the given information as shown above.

(See Topic 42)

Question from the 2008 FRM practice exam.

39. **D** Compared to an at-the-money European-styled call option, an out-of-the money European option with the same strike price and remaining maturity would have a greater negative value for theta.

A. Correct. Vega is highest for at-the-money options.
B. Correct. The delta for a European put option is negative, and the likelihood of exercise decreases, i.e., delta moves towards zero, as the price of the underlying stock increases.
C. Correct. Gamma increases as the time to maturity decreases. As time to maturity approaches zero, gamma approaches infinity.
D. Incorrect. Theta is large and negative for an at-the-money European-styled option, whilst theta is close to zero when the price for the underlying stock is very low. Therefore the theta for an out-of-the-money European styled call option would have a lower negative value compared to that of an at-the-money European-styled call option.

(See Topic 42)

Question from the 2008 FRM practice exam.

40. **B** I, II, and IV

I. Correct. The frequency of operational loss events is typically assumed to follow a Poisson distribution, but only with the caveat that actual loss events tend to be more correlated than those represented by the theoretical distribution, and investigating whether loss event correlations are too great to assume a Poisson distribution would be prudent.
II. Correct. Using convolution to aggregate severity and frequency distributions assumes independence, so one would want to check that these distributions are in fact uncorrelated.
III. Incorrect. Since the current approach uses a copula function to estimate the firm-wide operational loss distribution, the current approach does not assume independence of operational loss events between trading desks.
IV. Correct. Estimating loss distributions from historical data assumes that the data is both reliable and sufficient to generate an accurate estimate, and investigating the quality of the historical data would be necessary.

(See Topic 46)

Question from the 2008 FRM practice exam.

41. **A** It is used to evaluate the potential impact on portfolio values of unlikely, although plausible, events or movements in a set of financial variables.

A. Correct. It describes 'stress testing'.
B. Incorrect. It is not about 'stress testing'.
C. Incorrect. As B is incorrect.
D. Incorrect. As A is correct.

(See Topic 47)

Question from the 2008 FRM practice exam.

42. **D** A sound internal system uses at-the-point-in-time scoring for small-to-medium-sized companies and private firms and through-the-cycle scoring for large firms.

Explanation: The approaches are not compatible or directly comparable, and using the two approaches for different firms can yield highly inconsistent and misleading results.

(See Topic 50)

Question from the 2008 FRM practice exam.

43. **C** 18.5%

Explanation: To answer this question the test taker must understand transition matrices. The easiest way to determine the answer would be to make this a square matrix including default in initial state. Then self multiplying the matrix to get the two year transition matrix. We can also manually do the calculation;

After year 1 there is a 10% chance of default and 80% chance of still being B and a 10% chance of being an A. In year 2 there is a 10% chance of default if B rated (or 80% * 10% = 8%) and a 5% chance of default if A rated (10% * 5% = 0.5%). The total probability is therefore 18.5%.

Answer 'a' assumes just one year.
Answer 'b' ignores the probability of default after becoming A rated.
Answer 'd' simply adds two 10% (inappropriately does the second year probability).

(See Topic 50)

Book 4 Formulas

Valuation and Risk Models

spot rate: $z(t) = 2\left[\left(\frac{1}{d(t)}\right)^{\frac{1}{2t}} - 1\right]$

price of the security: $P = \frac{C_1}{(1+y)^1} + \frac{C_2}{(1+y)^2} + \frac{C_3}{(1+y)^3} + \ldots + \frac{C_N}{(1+y)^N}$

$\text{PV of a perpetuity} = \frac{C}{y}$

dollar value of a basis point: $\text{DV01} = |\text{price at YTM}_0 - \text{price at YTM}_1|$

amount of bonds needed to hedge: $\text{HR} = \frac{\text{DV01 (per \$100 of initial position)}}{\text{DV01 (per \$100 of hedging instrument)}}$

$\text{duration} = \frac{BV_{-\Delta y} - BV_{+\Delta y}}{2 \times BV_0 \times \Delta y}$

$\text{convexity} = \frac{BV_{-\Delta y} + BV_{+\Delta y} - 2 \times BV_0}{BV_0 \times \Delta y^2}$

percentage price change ≈ duration effect + convexity effect

$= [-\text{duration} \times \Delta y \times 100] + \left[\left(\frac{1}{2}\right) \times \text{convexity} \times (\Delta y)^2 \times 100\right]$

portfolio duration: $D_{\text{Port}} = \sum_{j=1}^{K} w_j \times D_j$

risk-neutral valuation:

$$U = \text{size of the up-move factor} = e^{\sigma\sqrt{t}}$$

$$D = \text{size of the down-move factor} = e^{-\sigma\sqrt{t}} = \frac{1}{e^{\sigma\sqrt{t}}} = \frac{1}{U}$$

$$\pi_u = \text{probability of an up move} = \frac{e^{rt} - D}{U - D}$$

$$\pi_d = \text{probability of a down move} = 1 - \pi_u$$

expected value: $E(S_T) = S_0 e^{\mu T}$

Black-Scholes-Merton Option Pricing Model:

$$c_0 = \left[S_0 \times N(d_1)\right] - \left[X \times e^{-R_f^c \times T} \times N(d_2)\right]$$

$$p_0 = \left\{X \times e^{-R_f^c \times T} \times \left[1 - N(d_2)\right]\right\} - \left\{S_0 \times \left[1 - N(d_1)\right]\right\}$$

where:

$$d_1 = \frac{\ln\left(\frac{S_0}{X}\right) + \left[R_f^c + \left(0.5 \times \sigma^2\right)\right] \times T}{\sigma \times \sqrt{T}}$$

$$d_2 = d_1 - \left(\sigma \times \sqrt{T}\right)$$

continuously compounded returns: $u_i = \ln\left(\frac{S_i}{S_{i-1}}\right)$

$$\text{delta} = \Delta = \frac{\partial c}{\partial s}$$

$$\text{portfolio delta} = \Delta_p = \sum_{i=1}^{n} w_i \Delta_i$$

gamma: $\Gamma = \frac{\partial^2 c}{\partial s^2}$

relationship among delta, theta, and gamma: $r\Pi = \Theta + rS\Delta + 0.5\sigma^2 S^2 \Gamma$

Taylor Series approximation (order two): $f(x) \approx f(x_0) + f'(x_0)(x - x_0) + \frac{1}{2} f''(x_0)(x - x_0)^2$

multifactor models:

$$S_{it} = \alpha_i + \beta_{1i}I_{1t} + \beta_{2i}I_{2t} + \beta_{3i}I_{3t} + \ldots + \varepsilon_{it}$$

where:
R_{it} = the return on stock i for period t
I_{jt} = the risk factor index j for period t
β_{ji} = the sensitivity of stock i's stock return to factor j
ε_{it} = the residual estimated through ordinary least squares (OLS) or some other regression technique

income-based models:

$$INC_{it} = \alpha_i + \beta_{1i}I_{1t} + \beta_{2i}I_{2t} + \beta_{3i}I_{3t} + \ldots + \varepsilon_{it}$$

where:
INC_{it} = the historical reported earnings for firm i for period t

expected loss = exposure × loss given default × probability of default

expected loss = EL_H = AE × LGD × EDF

unexpected loss: $UL = AE \times \sqrt{EDF \times \sigma^2{}_{LGD} + LGD^2 \times \sigma^2{}_{EDF}}$

adjusted exposure = OS + α × COM_U

Using the Cumulative Z-Table

Probability Example

Assume that the annual earnings per share (EPS) for a large sample of firms is normally distributed with a mean of \$5.00 and a standard deviation of \$1.50. What is the approximate probability of an observed EPS value falling between \$3.00 and \$7.25?

If EPS = x = \$7.25, then $z = (x - \mu)/\sigma = (\$7.25 - \$5.00)/\$1.50 = +1.50$

If EPS = x = \$3.00, then $z = (x - \mu)/\sigma = (\$3.00 - \$5.00)/\$1.50 = -1.33$

For z-value of 1.50: Use the row headed 1.5 and the column headed 0 to find the value 0.9332. This represents the area under the curve to the left of the critical value 1.50.

For z-value of –1.33: Use the row headed 1.3 and the column headed 3 to find the value 0.9082. This represents the area under the curve to the left of the critical value +1.33. The area to the left of –1.33 is 1 – 0.9082 = 0.0918.

The area between these critical values is 0.9332 – 0.0918 = 0.8414, or 84.14%.

Hypothesis Testing – One-Tailed Test Example

A sample of a stock's returns on 36 non-consecutive days results in a mean return of 2.0%. Assume the population standard deviation is 20.0%. Can we say with 95% confidence that the mean return is greater than 0%?

H_0: $\mu \leq 0.0\%$, H_A: $\mu > 0.0\%$. The test statistic = *z*-statistic = $\frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$
= (2.0 – 0.0) / (20.0 / 6) = 0.60.

The significance level = 1.0 – 0.95 = 0.05, or 5%.

Since this is a one-tailed test with an alpha of 0.05, we need to find the value 0.95 in the cumulative *z*-table. The closest value is 0.9505, with a corresponding critical *z*-value of 1.65. Since the test statistic is less than the critical value, we fail to reject H_0.

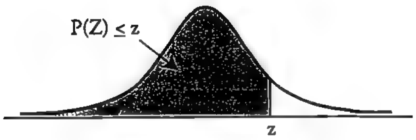
Hypothesis Testing – Two-Tailed Test Example

Using the same assumptions as before, suppose that the analyst now wants to determine if he can say with 99% confidence that the stock's return is not equal to 0.0%.

H_0: $\mu = 0.0\%$, H_A: $\mu \neq 0.0\%$. The test statistic (*z*-value) = (2.0 – 0.0) / (20.0 / 6) = 0.60.
The significance level = 1.0 – 0.99 = 0.01, or 1%.

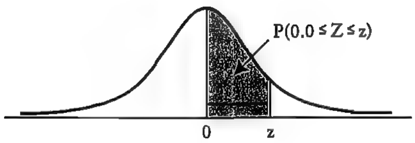
Since this is a two-tailed test with an alpha of 0.01, there is a 0.005 rejection region in both tails. Thus, we need to find the value 0.995 (1.0 – 0.005) in the table. The closest value is 0.9951, which corresponds to a critical *z*-value of 2.58. Since the test statistic is less than the critical value, we fail to reject H_0 and conclude that the stock's return equals 0.0%.

CUMULATIVE Z-TABLE



$P(Z \leq z) = N(z)$ for $z \geq 0$
$P(Z \leq -z) = 1 - N(z)$

z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.937	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.983	0.9834	0.9838	0.9842	0.9846	0.985	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.989
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.994	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990



ALTERNATIVE *Z*-TABLE

$P(Z \leq z) = N(z)$ for $z \geq 0$

$P(Z \leq -z) = 1 - N(z)$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3356	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4939	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

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